Mathematics for Modern Technology Homework

Instructions

1. Each assignment is to be done on one or more pieces of regular-sized notebook paper.
2. Your name and the assignment number should appear at the top of the first sheet.
3. Please do not use a red pen or red pencil to do the homework.
4. Please do not circle answers. Answer to a problem is not a single expression or number, it is the entire solution.
5. All relevant work is **required**. Problems are graded on the quality and correctness of the presented work.
6. Work through the homework problems referring to your notes and the lesson notes when necessary.
7. Redo the homework problems before an exam without referring to any other materials except for those that will be given to you on the exam. It is best to do this more than once.

1. Sets Homework Problems

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{2, 4, 6, 8, 10, 12, 14\}$
$B = \{2, 5, 8, 11, 14\}$
$C = \{1, 3, 6, 8, 11, 12, 14\}$

Find each of the following sets.

1. $A \cup B$

2. $A \cap B$

3. $A \cup B \cup C$

4. $A \cap B \cap C$

5. $\overline{A}$

6. $\overline{B}$

7. $\overline{C}$

8. $\overline{U}$

9. $\overline{A} \cap C$

10. $\overline{A} \cup C$

11. $\overline{B} \cap \overline{U}$

12. $\overline{A \cap B}$

13. $\overline{A} \cap \overline{B}$

14. $\overline{A \cup B}$

15. $\overline{A} \cup \overline{B}$

2. Inclusion-Exclusion Principle Homework Problems

1. Find $|A \cup B|$ if $|A| = 54$, $|B| = 39$, and $|A \cap B| = 17$.

2. Find $|A \cap B|$ if $|A| = 340$, $|B| = 68$, and $|A \cup B| = 372$.

3. Find $|B|$ if $|A \cup B| = 173$, $|A \cap B| = 23$, and $|A| = 95$.

4. A city neighborhood has 45 houses that are white, 67 houses that have two stories, and 16 houses which are white and have two stories. Use the inclusion-exclusion principle to determine the number of houses in the neighborhood that are either white or have two stories or both.

5. A car dealer has a group of 36 cars which are either blue or have cruise control or both. There are 25 blue cars and 27 cars which have cruise control. Use the inclusion-exclusion principle to determine how many of these cars are blue and have cruise control.

6. A gamers club has 123 members who play *Slackers of Doom* or *Library Adventure* or both games. If 87 of the gamers play *Library Adventure* and 45 of the gamers play both games, how many play *Slackers of Doom*?

7. Professor **X** has a class which has 35 students who can attend a tutoring session at either 1 pm on Saturday or 2 pm on Saturday or during both times. The tutoring session will be scheduled for the time during which the most students can attend. If 17 students can attend the 1 pm session and 13 students can attend both sessions, during which time will the tutoring session be scheduled?

8. A hotel has 100 rooms. There are 74 rooms which have a refrigerator, 60 rooms which have a microwave oven, and 47 rooms which have both a refrigerator and a microwave oven.

- **A.** Use the inclusion-exclusion principle to determine the number of rooms which have a refrigerator or a microwave oven or both.
- **B.** How many rooms have neither a refrigerator nor a microwave oven?
- **C.** How many rooms have a refrigerator but not a microwave oven?

9. During the month of May in city **Z**, it rained during 15 days, it was windy during 13 days, and it neither rained nor was it windy during 10 days.

- **A.** Determine the number of days in May during which it either rained or was windy or both in **Z**.
- **B.** Use the inclusion-exclusion principle to determine the number of days in May during which it both rained and was windy in **Z**.
- **C.** Determine the number of days in May during which it rained but was not windy in **Z**.

10. Assume $U = A \cup B$. Use the inclusion-exclusion principle to find $|A|$ if $\left|\overline{B}\right| = 52$, $|A \cup B| = 93$, and $|A \cap B| = 29$.

11. A small library has 70 titles that are either science fiction or fantasy or both. Of these, 17 titles are both science fiction and fantasy and 30 titles are not science fiction. Use the inclusion-exclusion principle to determine how many titles are fantasy.

12. Assume $U = A \cup B$. Use the inclusion-exclusion principle to find $|A \cup B|$ if $|A| = 50$, $|B| = 30$, and $\left|\overline{A \cap B}\right| = 60$.

3. Multiplication Principle Homework Problems

1. In how many ways can a president, a secretary, and a treasurer be elected from among ten people if no person may hold more than one office?

2. In how many ways can 4 boys and 6 girls be seated in a row if:

- **A.** There are no restrictions?
- **B.** One of the girls, Sue, must be seated on the left end?
- **C.** Boys sit together and girls sit together?

3. If 3 subjects are available for an 8:50 class and 5 other subjects for a 9:55 class, how many different class schedules are possible for a student who wishes to take one class at each time?

4. A quiz has 10 multiple choice questions and each question has 4 choices for its answer.

- **A.** How many different ways can the quiz be completed if every question is answered?
- **B.** How many different ways can the quiz be completed if zero or more questions are left unanswered?

5. Company **Z** makes 5 sizes of flat screens for flat panel televisions. Each size can be either an LCD screen or an LED screen; each screen has either a 60 hz screen refresh rate or a 120 hz screen refresh rate; and each screen has a vertical resolution of 720p or 1080p. How many different types of flat screens does **Z** make?

6. Every person has two biological parents. Use the multiplication principle to determine the number of biological great-great-great-great-grandparents that you have.

7.
- **A.** How many whole numbers have three digits?
- **B.** How many whole numbers have three digits and are even?
- **C.** How many whole numbers have three different digits?
- **D.** How many whole numbers have three different digits and are even?

8. Professor X is scheduling one hour individual Friday tutoring sessions for Ben, Nadine, Lisa, and Harold. The available times are 8:00 am, 9:00 am, 10:00 am, 11:00 am, 12:00 pm, and 1:00 pm.

- **A.** How many ways can Professor X schedule the sessions?
- **B.** How many ways can Professor X schedule the sessions if Ben's session must be at 11:00 am?
- **C.** How many ways can Professor X schedule the sessions if Nadine's session must be before Lisa's?

9. A traveler has a choice of 6 different routes between city **A** and city **B**, 2 different routes between city **B** and city **C**, 5 different routes between city **C** and city **D**, and 3 different routes between city **D** and city **E**.

- **A.** How many different routes may the traveler choose from when traveling from **A** to **E**?
- **B.** How many different routes may the traveler choose from when traveling from **A** to **D**?
- **C.** How many different routes may the traveler choose from when traveling from **B** to **E** and then back to **B**?
- **D.** Suppose the traveler travels from **A** to **E** and then travels back to **A** without using any of the routes used during the **A** to **E** trip. How many different routes may the traveler choose from for the **A** to **E** to **A** trip?

4. Permutations Homework Problems

1. **A.** How many arrangements are there of the letters in the word *computer*?

B. How many arrangements are there if only 3 letters of the word *computer* are used in each arrangement?

C. How many arrangements are there if only the first four letters of the word *computer* are used in each arrangement.

2. **A.** In how many different ways can seven books be lined up four at a time on a shelf?

B. A bookcase has two shelves and each shelf can hold 4 books. A book collection consists of 6 books about combinatorics and 5 books about topology. The top shelf of the bookcase will be filled with combinatorics books and the bottom shelf will be filled with topology books. How many different ways can books be arranged in the bookcase?

3. Six students are to receive awards for their outstanding performance during the past year. In how many different orders can the awards be distributed?

4. **A.** The governing council of the Abstract Mathematics Museum has 14 members. The council must select a chair, a secretary, and a treasurer to form the executive committee of the museum. If no person is permitted to hold more than one of the positions, how many ways can the three positions be filled?

B. The executive committee appoints the chair of the Topology Section and the chair of the Combinatorics Section. If neither chair can be a member of the executive committee and the chairs must be two different people, how many different ways can the chairs be appointed?

5. **A.** How many different ways can 3 girls and 3 boys be seated in a row of 6 chairs if boys must sit in the first 3 chairs?

B. How many different ways can 3 girls and 3 boys be seated in a row of 6 chairs if girls and boys alternate?

6. Crazy Pet Person owns 5 dogs and 4 cats. In order to annoy the neighbors, Crazy plans to line up 5 of the pets, with dogs and cats alternating, and lead them down the street in a parade. How many different ways can the line of pets be made?

5. Combinations Homework Problems

1. How many different ways can four goldfish be selected from a fish bowl containing 13 goldfish?

2. An art student is told to paint a picture using only 3 colors out of 24 available colors. How many different ways can the student select the three colors?

3. A committee of four is to be selected from a group containing eight workers and four bosses.

A. How many different ways can the committee be selected?

B. How many different ways can the committee be selected if the committee contains only workers?

C. How many different ways can the committee be selected if the committee contains only bosses?

D. How many different ways can the committee be selected if exactly one boss is on the committee?

E. How many different ways can the committee be selected if two bosses and two workers are on the committee?

4. A bag contains 10 balls which are numbered with the integers 0 through 9. Each ball has exactly one integer and each integer appears on exactly one ball. Three balls are to be selected from the bag.

A. How many different ways can the three balls be selected?

B. How many different ways can the three balls be selected if zero is not on any of the selected balls?

C. How many different ways can the three balls be selected if zero is on one of the selected balls?

D. How many different ways can the three balls be selected if the integers on the selected balls are even?

E. How many different ways can the three balls be selected if the integers on the selected balls are odd?

F. How many different ways can the three balls be selected if the three integers on the selected balls are even or the three integers on the selected balls are odd?

G. How many different ways can the three balls be selected if one of the selected balls has an even integer and two of the selected balls have odd integers?

5. In how many ways can a five-card hand be dealt from a 52-card deck if

A. All the cards are clubs?

B. Exactly two are clubs, two are diamonds, and one is a spade?

C. All are to be hearts or all are to be black cards?

6. In how many ways can a three-card hand be dealt from a 52-card deck if

A. All the cards are red?

B. All the cards are face cards?

C. At least one card is a face card?

7. If a coin is flipped seven times, how many different ways can the result be four heads and three tails?

6. Probability Homework Problems

1. An experiment consists of rolling a die and tossing a quarter.

A. Use the multiplication principle to determine the number of outcomes of the experiment.

B. Write the sample space of the experiment.

C. Assume that the die and quarter are unbiased so that the outcomes are equally likely when a trial of the experiment is run. Write the probability distribution of the experiment. Write each probability as a fraction.

D. How many outcomes in the sample space have three for the die roll?

E. Use the result of the previous part to compute the probability that the die roll is three. Write the probability as a fraction.

F. How many outcomes in the sample space have the quarter toss as tails?

G. Use the result of the previous part to compute the probability that the quarter toss is tails. Write the probability as a fraction.

H. How many outcomes in the sample space have a die roll that is less than 5?

I. Use the result of the previous part to compute the probability that the die roll is less than five. Write the probability as a fraction.

J. How many outcomes in the sample space have an even die roll?

K. Use the result of the previous part to compute the probability that the die roll is even. Write the probability as a fraction.

L. How many outcomes in the sample space have six for the die roll or tails for the quarter toss?

M. Use the result of the previous part to compute the probability that the die roll is a six or the quarter toss is tails. Write the probability as a fraction.

N. How many outcomes in the sample space have an even die roll or heads for the quarter toss?

O. Use the result of the previous part to compute the probability that the die roll is even or the quarter toss is heads. Write the probability as a fraction.

P. How many outcomes in the sample space have a die roll that is less than 5 or heads for the quarter toss?

Q. Use the result of the previous part to compute the probability that the die roll is less than 5 or the quarter toss is heads. Write the probability as a fraction.

2. An experiment consists of tossing a penny, a nickel, a dime, and a quarter.

A. Use the multiplication principle to determine the number of outcomes of the experiment.

B. Write the sample space of the experiment.

C. Assume that the coins are unbiased so that the outcomes are equally likely when a trial of the experiment is run. Write the probability distribution of the experiment. Write each probability as a fraction.

D. How many outcomes in the sample space have tails for the nickel toss?

E. Use the result of the previous part to compute the probability that the nickel toss is tails. Write the probability as a fraction.

F. How many outcomes in the sample space have heads for the penny toss or tails for the quarter toss?

G. Use the result of the previous part to compute the probability that the penny toss is heads or the quarter toss is tails. Write the probability as a fraction.

H. How many outcomes in the sample space have no heads?

I. Use the result of the previous part to compute the probability that none of the tosses are heads. Write the probability as a fraction.

J. How many outcomes in the sample space have exactly one head?

K. Use the result of the previous part to compute the probability that exactly one of the tosses is heads. Write the probability as a fraction.

L. How many outcomes in the sample space have exactly two heads?

M. Use the result of the previous part to compute the probability that exactly two of the tosses are heads. Write the probability as a fraction.

N. How many outcomes in the sample space have exactly three heads?

O. Use the result of the previous part to compute the probability that exactly three of the tosses are heads. Write the probability as a fraction.

P. How many outcomes in the sample space have four heads?

Q. Use the result of the previous part to compute the probability that all four of the tosses are heads. Write the probability as a fraction.

R. How many outcomes in the sample space have at least two tails?

S. Use the result of the previous part to compute the probability that at least two of the tosses are tails. Write the probability as a fraction.

T. How many outcomes in the sample space have at most three tails?

U. Use the result of the previous part to compute the probability that at most three of the tosses are tails. Write the probability as a fraction.

V. How many outcomes in the sample space have exactly three tails or exactly two heads?

W. Use the result of the previous part to compute the probability that at exactly three of the tosses are tails or exactly two of the tosses are heads. Write the probability as a fraction.

3. Each of a group of 25 students were asked how many candy bars they had eaten in the last week.

7 ate 0 candy bars
10 ate 1 candy bar
6 ate 2 candy bars
2 ate 3 candy bars

A. The probability of an outcome is its relative frequency. Write the probability distribution. Write each probability as a decimal to two decimal places.

B. What is the probability that a student ate either no candy bars or three candy bars? Write the probability as a decimal to two decimal places.

C. What is the probability that a student ate an odd number of candy bars? Write the probability as a decimal to two decimal places.

D. What is the probability that a student ate at least one candy bar? Write the probability as a decimal to two decimal places.

E. What is the probability that a student ate at most two candy bars? Write the probability as a decimal to two decimal places.

F. What is the probability that a student ate either at least one candy bar or at most two candy bars?

7. Probability Computations Homework Problems

1. A bag contains 6 red balls, 9 green balls, and 4 yellow balls. Five balls are selected from the bag. Assume that all of the different ways of selecting five balls from the bag are equally likely of being selected.

A. What is the probability of selecting 4 yellow balls? Write the probability as a rational number which has one in the numerator.

B. What is the probability of selecting no yellow balls? Write the probability as a rational number which has one in the numerator.

C. What is the probability of selecting no green balls? Write the probability as a rational number which has one in the numerator.

D. What is the probability of selecting 2 red balls and 3 green balls? Write the probability as a rational number which has one in the numerator.

E. What is the probability of selecting 2 red balls? Write the probability as a rational number which has one in the numerator.

F. What is the probability of selecting 3 green balls? Write the probability as a rational number which has one in the numerator.

G. What is the probability of selecting 2 red balls or 3 green balls? Write the probability as a rational number which has one in the numerator.

2. A group of people contains 6 women and 5 men. A committee of 4 is to be selected from this group. Assume that the committees are equally likely.

A. What is the probability of choosing exactly 3 women for the committee? Write the probability as a rational number which has one in the numerator.

B. What is the probability of choosing exactly 3 men for the committee? Write the probability as a rational number which has one in the numerator.

C. What is the probability of choosing exactly 2 women for the committee? Write the probability as a rational number which has one in the numerator.

D. Sue is one of the women in the group. What is the probability of choosing Sue to be on the committee? Write the probability as a rational number which has one in the numerator.

E. Fred and Barney are two of the men in the group. What is the probability of choosing Fred and Barney to be on the committee? Write the probability as a rational number which has one in the numerator.

F. What is the probability of choosing Fred to be on the committee? Write the probability as a rational number which has one in the numerator.

G. What is the probability of choosing Barney to be on the committee? Write the probability as a rational number which has one in the numerator.

H. What is the probability of choosing either Fred or Barney to be on the committee? Write the probability as a rational number which has one in the numerator.

3. A five-card hand is to be selected from a 52-card deck. Assume that all of the different ways of selecting a five-card hand from the deck are equally likely of being selected.

A. What is the probability that all of the cards are diamonds? Write the probability as a rational number which has one in the numerator.

B. What is the probability that all of the cards come from the same suit? Write the probability as a rational number which has one in the numerator.

C. What is the probability that all of the cards are black? Write the probability as a rational number which has one in the numerator.

D. What is the probability that none of the cards are hearts? Write the probability as a rational number which has one in the numerator.

E. What is the probability that exactly three of the cards are diamonds? Write the probability as a rational number which has one in the numerator.

F. What is the probability that exactly two of the cards are threes? Write the probability as a rational number which has one in the numerator.

G. What is the probability that none of the cards is a jack? Write the probability as a quotient which has one in the numerator and a denominator rounded to the nearest 1/10th.

8. Random Variables Homework Problems

1. A math class was given a 5-point quiz. Let X be the random variable that denotes the score of a student taking the quiz. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
0	4
1	7
2	19
3	16
4	8
5	2

A. Write the probability distribution of X. Write each probability as a decimal to two decimal places.

B. Draw a histogram to represent the probability distribution graphically.

C. Determine the probability that a student taking the quiz scored at least 3.

2. A group of slackers were asked how many hours each day they spend playing video games. Let X be the random variable that denotes the number of hours per day a slacker plays video games. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
0	2
1	14
2	20
3	42
4	25
5	6
6	7
7	3
8	1

A. Write the probability distribution of X. Write each probability as a decimal to two decimal places.

B. Draw a histogram to represent the probability distribution graphically.

C. Determine the probability that a slacker plays video games for at most 3 per day.

3. A penny, nickel, dime and quarter are to be flipped. Let X be the random variable that denotes the number of heads that occur when the four coins are flipped.

A. Write the frequency distribution of X.

B. Write the probability distribution of X. Write each probability as a decimal to two decimal places.

C. Draw a histogram to represent the probability distribution graphically.

D. Determine the probability of getting at least two heads when the four coins are flipped.

4. A two-card hand is drawn from a 52-card deck. If both cards are face cards, the hand counts as 3 points. If both cards are in the same suit and at most one is a face card, the hand counts as 2 points. If one card is a heart and the other a diamond, or one card is a club and the other is a spade, and the hand has at most one face card, the hand counts as one point. If the hand is none of the above types, the hand counts as zero points. Let X be the random variable that denotes the number of points a two-card hand is worth when it is drawn from the deck. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
0	640
1	320
2	300
3	66

A. Write the probability distribution of X. Write each probability as a decimal to two decimal places.

B. Draw a histogram to represent the probability distribution graphically.

C. Determine the probability that a two-card hand drawn from the deck is worth either zero points or three points.

9. Expected Value Homework Problems

1. The exam scores for two classes are given below.

Class X			Class Y		
57	63	29	50	44	44
85	93	43	80	94	94
99	90	80	74	78	68
71	59	53	37	86	74
60	60	84	87	89	93

A. Determine the mean score for class **X**. Round the mean score to the nearest integer.

B. Determine the mean score for class **Y**. Round the mean score to the nearest integer.

C. Determine the mean score for both classes put together. Round the mean score to the nearest integer.

D. Use the results of the previous parts to decide which class did better on the exam. Explain your answer.

2. The daily high temperatures (in degrees Fahrenheit) for thirty consecutive winter days in city **Z** are given below.

31.1	25.2	31.0	25.9	23.5	31.1	32.3	38.6	31.6	44.5
28.5	29.2	24.5	30.9	27.2	28.2	28.0	24.5	25.0	27.0
34.5	36.6	28.7	31.9	22.7	27.5	25.1	29.3	30.3	33.1

A. Compute the mean daily high temperature. Round the mean to the nearest 1/10th of a degree.

B. Determine the number of days during which the daily high temperature was above the mean.

C. Determine the number of days during which the daily high temperature was below the mean.

3. A group of students were asked how many different computer games they play. Let X be the random variable that denotes the number of different computer games played by a student. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
0	19
1	13
2	7
3	2
4	6
5	3
6	1

A. Compute the mean number of video games regularly played by the members of the group. Round the mean to the nearest 1/10th.

B. Is it possible for a student to actually play the mean number of video games? Explain your answer.

C. Determine the number of students who regularly play more than the mean number of video games.

D. Determine the percentage of students who regularly play more then the mean number of video games.

4. Q made many phone calls during the month of March, but none were longer than six minutes in length. Let X be the random variable that denotes the length of a phone call made by Q. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
1	21
2	18
3	0
4	39
5	62
6	10

A. How many phone calls did Q make during March?

B. Compute the mean length of a phone call. Round the mean to the nearest 1/10th of a minute.

C. If Q is billed an extra \$0.10 for every call that lasts longer than the mean, how much extra was Q billed for March?

5. Let X be a random variable which denotes the number of computers owned by the members of a Linux users group. The probability distribution of X is shown below.

x	$P(X = x)$
1	0.22
2	0.37
3	0.19
4	0.08
5	0.10
6	0.02
7	0.02

A. Determine the mean number of computers owned by members of the users group. Round the mean to the nearest 1/10th.

B. Determine the percentage of members of the user group who own fewer than the mean number of computers.

6. There are 30 students enrolled in a section of a course. Let X be a random variable which denotes the number of students who attend an entire class meeting of the section. The probability distribution of X is shown below.

x	$P(X = x)$	x	$P(X = x)$
20	0.06	26	0.18
21	0.07	27	0.15
22	0.05	28	0.08
23	0.09	29	0.04
24	0.10	30	0.05
25	0.13		

A. Determine the mean number of students who attend an entire class. Round the mean to the nearest 1/10th of a student.

B. On average, what percentage of the students taking the course attend an entire class?

7. A candy manufacturer makes red, green, and yellow candy pieces that are mixed together in packages. The mean number of green candy pieces in a package is 19%. If a package contains 46 candy pieces, then how many are expected to be red or yellow?

8. A game requires a player to draw one card from a 52-card deck. If the card is an ace or a king, the player wins \$3. If the card is a queen, jack, or ten, the player wins \$1. Otherwise the player loses \$1.50. Let random variable X denote the amount of money a player wins after drawing a card from the deck.

- **A.** Write the probability distribution of X. Write each probability as a rational number (a fraction).
- **B.** Compute the expected value of X.
- **C.** If the player plays the game 100 times, how much money is the player expected to lose?
- **D.** How much money should the owner of the game expect to earn per game?

9. A player rolls a pair of dice. If their sum is 6 or less, the player wins \$9. Otherwise the player loses \$7. Let random variable X denote the amount of money a player wins after rolling the pair of dice.

- **A.** Write the probability distribution of X. Write each probability as a rational number (a fraction).
- **B.** Compute the expected value of X.
- **C.** If the player plays the game 100 times, how much money is the player expected to win?
- **D.** How much money should the owner of the game expect to earn per game?

10. A math exam has 5 true-false questions, and 6 multiple-choice questions with four choices each.

- **A.** Let random variable X denote the number of correct answers selected by a student taking the exam and guessing at the answer to each question. Determine the expected value of X.
- **B.** Is it likely that a student who guesses the answer to each question will pass the exam? Explain your answer.

11. The **Z** company wishes to offer a replacement plan for its low-cost laptop computers. It costs the company \$500 to replace a laptop and there is a 1% probability that a laptop will need to be replaced after it is purchased. The company does not wish to offend its customers by making a profit on the replacement plan, but it doesn't want to lose money on the plan either. How much should **Z** charge a customer willing to buy a replacement plan policy?

10. Standard Deviation Homework Problems

1. The exam scores for a course and the two classes taking the course are given below.

Course						**Class X**			**Class Y**		
57	63	29	50	44	44	57	63	29	50	44	44
85	93	43	80	94	94	85	93	43	80	94	94
99	90	80	74	78	68	99	90	80	74	78	68
71	59	53	37	86	74	71	59	53	37	86	74
60	60	84	87	89	93	60	60	84	87	89	93

- **A.** Compute μ and σ for the course. Round each statistic to the nearest 1/10th.
- **B.** If a score greater than $\mu - \sigma$ is passing, how many students in the course passed the exam?
- **C.** If a score greater than $\mu + \sigma$ is an A, how many students in the course received an A on the exam?
- **D.** Compute μ and σ for class **X**. Round each statistic to the nearest 1/10th.
- **E.** Compute μ and σ for class **Y**. Round each statistic to the nearest 1/10th.

2. **A.** The daily high temperatures (in degrees Fahrenheit) for thirty consecutive winter days in city **Z** are given below. Compute the mean and standard deviation of this data. Round each statistic to the nearest 1/10th.

31.1	25.2	31.0	25.9	23.5	31.1	32.3	38.6	31.6	44.5
28.5	29.2	24.5	30.9	27.2	28.2	28.0	24.5	25.0	27.0
34.5	36.6	28.7	31.9	22.7	27.5	25.1	29.3	30.3	33.1

B. We say that data value x is *within one standard deviation of the mean* if $\mu - \sigma \leq x \leq \mu + \sigma$. For example, if the mean is 100 and the standard deviation is 10, then data value x is within one standard deviation of the mean if $100 - 10 \leq x \leq 100 + 10$ $(90 \leq x \leq 110)$. Similarly, if μ is the mean, σ the standard deviation, n a positive integer, and x a data value, then x will be within n standard deviations of the mean if $\mu - n \cdot \sigma \leq x \leq \mu + n \cdot \sigma$. (We say that σ is *one standard deviation* and that $n \cdot \sigma$ is *n standard deviations*.) How many of the daily high temperatures are within one standard deviation of the mean?

C. What percentage of the data values are within one standard deviation of the mean?

D. A sample of the above data is listed below. Compute the mean and sample standard deviation of the sample. Round each statistic to the nearest 1/10th.

31.1	25.2	31.0	25.9	23.5	31.1	32.3	38.6
34.5	36.6	28.7	31.9	22.7	27.5	25.1	29.3

E. How many of the sample's data values are within two standard deviations of its mean?

F. What percentage of the sample's data values are within two standard deviations of its mean?

3. A group of students were asked how many different computer games they play. Let X be the random variable that denotes the number of different computer games played by a student. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
0	19
1	13
2	7
3	2
4	6
5	3
6	1

A. Compute the mean and standard deviation of X. Round each statistic to the nearest 1/10th.

B. A student is considered to be obsessed with computer games if the number of different computer games the student plays is more than one standard deviation above the mean. How many of the students are obsessed with computer games?

C. Determine the percentage of students obsessed with computer games.

4. Q made many phone calls during the month of March, but none were longer than six minutes in length. Let X be the random variable that denotes the length of a phone call made by Q. The frequency distribution of X is given below (x denotes a value of X).

x	**Frequency**
1	21
2	18
3	0
4	39
5	62
6	10

A. Compute the mean and standard deviation of X. Round each statistic to the nearest 1/10th.

B. How many phone calls did Q make during the five days which had lengths that were more than one standard deviation from the mean?

C. During most normal five day periods, around 68% of the phone calls Q makes have lengths within one standard deviation of the mean. Would this five day period be considered normal? Explain your answer.

5. Let X be a random variable which denotes the number of computers owned by the members of a Linux users group. The probability distribution of X is shown below.

x	$P(X = x)$
1	0.22
2	0.37
3	0.19
4	0.08
5	0.10
6	0.02
7	0.02

A. Compute the mean and standard deviation of X. Round each statistic to the nearest 1/10th.

B. What is the probability that the number of computers a member of the group owns is within one standard deviation of the mean?

6. There are 30 students enrolled in a section of a course. Let X be a random variable which denotes the number of students who attend an entire class meeting of the section. The probability distribution of X is shown below.

x	$P(X = x)$	x	$P(X = x)$
20	0.06	26	0.18
21	0.07	27	0.15
22	0.05	28	0.08
23	0.09	29	0.04
24	0.10	30	0.05
25	0.13		

A. Compute the mean and standard deviation of X. Round each statistic to the nearest 1/10th.

B. What is the probability that the number of students attending an entire class is fewer than one standard deviation below the mean?

11. Normal Distribution Homework Problems

1. The serum cholesterol levels of residents of country **Q** aged 20 and over are normally distributed with a mean of 202 mg/dL and a standard deviation of 40.

A. Determine the probability that a resident of **Q** has a cholesterol level in the 180–220 range.

B. Determine the probability that a resident of **Q** has a cholesterol level that is 240 or higher. A cholesterol level that is 240 or higher is considered to be high.

C. Determine the probability that a resident of **Q** has a cholesterol level in the 150–190 range.

D. In a random sample of 250,000 residents of **Q**, how many would be expected to have a cholesterol level in the 180–220 range? How many would be expected to have a cholesterol level that is 240 or higher? How many would be expected to have a cholesterol level in the 150–190 range?

2. A manufacturer of compact fluorescent light bulbs determines that the life span of the bulbs is normally distributed with a mean of 5 years and a standard deviation of 6 months.

A. If the light bulbs are guaranteed for 4 years after the date of purchase, what is the probability that a bulb burns out before the warranty expires?

B. If 4 million bulbs are sold, how many are expected to burn out before the warranty expires?

3. The yearly rainfall in city **J** is normally distributed with a mean of 47 inches and a standard deviation of 10 inches.

A. Determine the probability that **J** will receive more than 70 inches of rain during a year.

B. If **Z** receives fewer than 30 inches of rain during a year, then the city has difficulty meeting its water consumption needs during that year. How many years per century should the city expect to have difficulty meeting its water consumption needs?

C. The city's water system functions best during years it receives between 40 and 55 inches of rain. Determine the probability that the city's water system will function at its best during the next year.

4. It takes K an average of 30 minutes to drive to work with a standard deviation of 10 minutes. Assume that K's drive to work times are normally distributed. If it takes more than 40 minutes to drive to work, then K is late to work. K works 5 days per week for 50 weeks each year. Estimate the number of days K is late to work each year.

5. An orange grove produces ripe oranges that have a mean circumference of 11 inches with a standard deviation of 0.75 inches. Assume that the circumferences of the ripe oranges produced by the orange grove are normally distributed.

A. If a ripe orange is less than 9.75 inches in circumference, then the orange grove throws it away. Determine the probability that a ripe orange is thrown away by the orange grove.

B. If a ripe orange is greater than 12 inches in circumference, then it is sold as a premium orange. Determine the probability that a ripe orange is a premium orange.

C. Ripe oranges that are between 9.75 and 12 inches in circumference are sold as standard oranges. Standard oranges are packaged in bags containing a dozen oranges each. Estimate the number of bags of standard oranges the orange grove produces from a crop of 50,000 ripe oranges.

12. Statements and Truth Tables Homework Problems

1. Use the following statements to write each statement in sentence form.

p: Ursula is a plumber.

q: Ursula is an electrician.

A. $\neg p$

B. $p \vee q$

C. $\neg p \wedge q$

D. $\neg p \vee \neg q$

2. Use the following statements to write each statement in symbolic form.

p: The cat jumped onto the table.

q: The cat ate the mouse.

A. The cat did not eat the mouse.

B. The cat did not jump onto the table, or it ate the mouse.

C. The cat did not eat the mouse, but it jumped onto the table.

D. The cat did not jump onto the table, however it did eat the mouse.

3. Write the truth table for each of the following statements.

A. $p \vee \neg p$

B. $p \wedge \neg p$

C. $\neg p \wedge (\neg q \vee q)$

D. $(p \vee \neg q) \wedge \neg p$

E. $(p \wedge \neg r) \vee (q \wedge \neg p)$

13. Implications Homework Problems

1. Use the following statements to write each statement in sentence form.

p: Marley owns a guitar.

q: Marley is in a band.

r: Marley plays bass.

A. $p \to q$

B. $(\neg q) \to (\neg p)$

C. $(\neg q) \to (\neg p \vee \neg r)$

D. $(\neg r \wedge \neg p) \to (\neg q)$

2. Use the following statements to write each statement in symbolic form.

p: Rita sets the table.

q: Henry makes the salad.

r: Julia pours the drinks.

A. If Rita sets the table, then Henry makes the salad and Julia pours the drinks.

B. Julia does not pour the drinks if Henry does not make the salad.

C. Julia pours the drinks only if Henry makes the salad and Rita does not set the table.

D. Henry making the salad the salad is a necessary condition for Rita to set the table or Julia to pour the drinks.

E. Rita not setting the table or Julia not pouring the drinks is a sufficient condition for Henry to not make the salad.

3. For each statement, identify the hypothesis and the conclusion, and write the converse as a sentence. The converse must be written in the "if p, then q" form.

A. If the dog has no fleas, then it is happy.

B. The bridge is closed only if its surface is icy.

C. Students do not show up for the class if the professor is not interesting or the class is cancelled.

D. A natural number's last digit being 5 is a sufficient condition for it to be an odd number.

E. The polygon having four sides is a necessary condition for it to be a rectangle.

4. Write the truth table for each of the following statements.

A. $(\neg q) \to (\neg p)$

B. $(\neg p) \to (p \vee \neg q)$

C. $(p \vee q) \to (p \wedge q)$

D. $(p \wedge r) \to (\neg q)$

E. $(\neg p) \leftrightarrow (q \vee p)$

5. Determine if each statement is true or false. Justify your answers.

A. If $1 + 1 = 2$, then $1 + 1 = 0$.

B. If $1 + 1 = 0$, then $1 + 1 = 2$.

C. It is the case that $1 + 1 = 0$ only if $1 + 1 = 2$.

D. It is the case that $1 + 1 = 0$ if $1 + 1 = 2$.

E. It is the case that $1 + 1 = 2$ is a sufficient condition for $1 + 1 = 0$.

F. It is the case that $1 + 1 = 2$ is a necessary condition for $1 + 1 = 0$.

G. $1 + 1 = 0$ if and only if $1 + 1 = 2$.

14. Logical Equivalence Homework Problems

1. Why is $(p \wedge q) \leftrightarrow (q \to p)$ not a tautology?

2. Show that $(p \to q) \Leftrightarrow (\neg p \vee q)$.

3. Write the sentence form of the negation of each of the following statements.

A. The road was icy and the traffic was heavy.

B. The game was close and exciting.

C. Emmy ate a slice of pizza and she did not eat all of it.

D. The child was playing or she was reading a book.

E. Either Rita or Henry will clean off the table.

F. The professor was interesting or his students failed to show up for class.

4. For each of the following statements, write the contrapositive and the negation in sentence form.

A. If the leaves are falling off of the tree, then it is October.

B. If the day is sunny, then it is not raining.

C. The dog has fleas only if it is not happy.

D. Newton does not own a cat if he owns a pet.

E. Garcia is either reading email or playing a video game if he is using his computer.

15. Quantifiers Homework Problems

1. All lions have sharp teeth.

2. Some dinosaurs smoked cigars.

3. All vegetarians do not eat meat.

4. Some of the students in the class do not have good grades.

5. At most five of the students in the class did not pass the exam.

6. At least nine members of the group of people are happy.

7. The number of junkers on the used car lot is at most 20.

8. The hand of cards has at most two aces.

9. Aliens have visted the planet at least seven times during the past week.

10. No person has curly hair.

11. Every person has curly hair.

12. The number of people who have 11 fingers is at least 1000.

13. Tilly has been to Hawaii at most two times.

14. It never rains in the desert.

15. It always rains in a rain forest.

16. Compound Interest Homework Problems

1. **A.** A certificate of deposit account has an interest rate of 8.4%. If the interest on the account is compounded monthly, what is the account's monthly interest rate?

B. If the account has a balance of $1240.72 at the beginning of a month, what is the balance at the end of that month?

C. If the account has a balance of $3254.25 at the end of a month, how much interest was earned during that month?

2. **A.** The interest on a certificate of deposit account is compounded quarterly and the account earns 1.5% interest every quarter. If the account has a balance of \$11,820.42 at the beginning of a year, what is the balance after 9 months of that year have passed?

B. What is the account's interest rate?

3. If \$1500 is deposited into a certificate of deposit account which has a 6.8% interest rate, how much is in the account after 7 years if the interest is compounded

A. quarterly?

B. monthly?

C. daily?

4. If \$16,500 is needed in 5 years, what amount should be deposited now into a certificate of deposit account that earns 7.3% compounded

A. quarterly?

B. monthly?

C. daily?

5. A certificate of deposit account which earns 3.2% compounded monthly will have a balance of \$10,000 after 3 years. What will the balance be after 6 years?

6. Ten years ago, an investor deposited some funds into a certificate of deposit account that earns 4.75% compounded weekly. The balance is now \$17,000. What was the balance 4 years after the initial deposit?

7. **A.** What is the effective rate if an account earns 5.70% compounded quarterly? Round the effective rate to the nearest 1/100th of a percent.

B. What is the effective rate if an account earns 5.70% compounded monthly? Round the effective rate to the nearest 1/100th of a percent.

C. What is the effective rate if an account earns 5.70% compounded daily? Round the effective rate to the nearest 1/100th of a percent.

8. Is it better for a certificate of deposit account to earn 15.00% compounded daily or 15.10% compounded monthly?

Finance Formulas

Compound Interest

$r =$ the annual interest rate
$m =$ the number of interest payments per year
$n =$ the total number of interest payments
$P =$ the principal
$F =$ the compound amount (balance)
$r_{\text{eff}} =$ the effective rate

$$F = P\left(1 + \frac{r}{m}\right)^n$$

$$P = \frac{F}{\left(1 + \frac{r}{m}\right)^n}$$

$$r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$$

17. Annuities Homework Problems

1. **A.** An investor will deposit \$250 each month into an account that pays 6.5% compounded monthly. How much will be in the account in 25 years?

B. What is the total amount that the investor pays into the account?

2. **A.** Fred's daughter Pebbles will be 18 soon. When Pebbles turns 18, Fred plans to deposit enough money into an account that pays 9% compounded semiannually so that Pebbles may withdraw \$3150 every six months for four years. How much should Fred deposit into the account?

B. How much money does Pebbles get from the account?

3. **A.** An investor wishes to make monthly equal payments into an account that pays 8.1% compounded monthly. What is the amount of one payment if the investor wishes to have \$250,000 in the account in 35 years?

B. How much money does the investor pay into the account?

4. **A.** Barney deposits \$85,000 into an account that pays 7.4% compounded quarterly. Barney plans to withdraw from the account equal amounts at the end of each quarter for six years, leaving nothing in the account at the end. How much may Barney withdraw at the end of each quarter?

B. What is the total amount that Barney withdraws from the account?

5. **A.** City **Z** must repay some bond holders 2.5 million dollars in 15 years. In order to raise that amount, the city plans to deposit equal payments every 6 months into an account that earns 4.3% compounded semiannually. How much is one payment?

B. What is the total amount the city deposits into the account?

6. A lottery winner has the option of taking either a lump sum payment or taking annual payments of 10 million dollars at the end of each year for 20 years. If the winner chooses the annual payments, then the lottery deposits the lump sum into an account which pays 6% compounded annually. The annual payments will be taken out of the account and given to the winner at the end of each year. How much is the lump sum?

7. **A.** On December 31 of the year when Sheila is age 8, her parents deposit \$5000 into a trust fund that pays 4% compounded annually. On December 31 when she is 21, 22, and 23 years old, Sheila will receive X dollars from the trust fund. The trust fund will be empty when Sheila receives the third payment. Determine X.

B. How much does Sheila receive from the trust fund?

Finance Formulas

Compound Interest

$r =$ the annual interest rate
$m =$ the number of interest payments per year
$n =$ the total number of interest payments
$P =$ the principal
$F =$ the compound amount (balance)
$r_{\text{eff}} =$ the effective rate

$$F = P\left(1 + \frac{r}{m}\right)^n$$

$$P = \frac{F}{\left(1 + \frac{r}{m}\right)^n}$$

$$r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$$

Increasing Annuities

$r =$ the annual interest rate
$m =$ the number of payments per year
$n =$ the total number of payments
$p =$ the amount of one payment
$F =$ the future value

$$F = \left[\frac{mp}{r}\right]\left[\left(1+\frac{r}{m}\right)^n - 1\right]$$

$$p = \frac{rF}{m\left[\left(1+\frac{r}{m}\right)^n - 1\right]}$$

Decreasing Annuities

$r =$ the annual interest rate
$m =$ the number of withdrawals per year
$n =$ the total number of withdrawals
$p =$ the amount of one withdrawal
$PV =$ the present value

$$PV = \left[\frac{mp}{r}\right]\left[1 - \frac{1}{\left(1+\frac{r}{m}\right)^n}\right]$$

$$p = \frac{rPV\left(1+\frac{r}{m}\right)^n}{m\left[\left(1+\frac{r}{m}\right)^n - 1\right]}$$

18. Amortization Homework Problems

1. A buyer borrows $15,000 at 11.3% compunded monthly for 30 months in order to purchase a car.

A. How much is one payment?

B. What is the unpaid balance after 15 months?

C. How much interest will be paid during the first 15 months?

D. How much interest will be paid during the final 15 months?

E. How much of the first payment is applied to the principal?

F. How much of the last payment is applied to the principal?

2. A buyer purchases retail space for $750,000. The buyer takes out a 15-year mortgage at 6.5% compounded monthly to finance the purchase.

A. How much is one payment?

B. How much interest is paid with the 90th payment?

C. How much of the 90th payment is applied to the principal?

D. How much is applied to the principal during the first year?

E. How much interest is paid during the first year?

F. How much is applied to the principal during the 14th year?

G. How much interest is paid during the 14th year?

Finance Formulas

Compound Interest

$r =$ the annual interest rate
$m =$ the number of interest payments per year
$n =$ the total number of interest payments
$P =$ the principal
$F =$ the compound amount (balance)
$r_{\text{eff}} =$ the effective rate

$$F = P\left(1+\frac{r}{m}\right)^n$$

$$P = \frac{F}{\left(1+\frac{r}{m}\right)^n}$$

$$r_{\text{eff}} = \left(1+\frac{r}{m}\right)^m - 1$$

Increasing Annuities

$r =$ the annual interest rate
$m =$ the number of payments per year
$n =$ the total number of payments
$p =$ the amount of one payment
$F =$ the future value

$$F = \left[\frac{mp}{r}\right]\left[\left(1+\frac{r}{m}\right)^n - 1\right]$$

$$p = \frac{rF}{m\left[\left(1+\frac{r}{m}\right)^n - 1\right]}$$

Decreasing Annuities

$r =$ the annual interest rate
$m =$ the number of withdrawals per year
$n =$ the total number of withdrawals
$p =$ the amount of one withdrawal
$PV =$ the present value

$$PV = \left[\frac{mp}{r}\right]\left[1 - \frac{1}{\left(1+\frac{r}{m}\right)^n}\right]$$

$$p = \frac{rPV\left(1+\frac{r}{m}\right)^n}{m\left[\left(1+\frac{r}{m}\right)^n - 1\right]}$$

Amortization

$r =$ the annual interest rate
$m =$ the number of payments per year
$n =$ the total number of payments
$p =$ the amount of one payment
$PV =$ the principal
$B_k =$ the unpaid balance after k payments
$P_k =$ the principal paid with the kth payment
$I_k =$ the interest paid with the kth payment

$$p = \frac{rPV\left(1+\frac{r}{m}\right)^n}{m\left[\left(1+\frac{r}{m}\right)^n - 1\right]}$$

$$B_k = \left[\frac{mp}{r}\right]\left[1 - \frac{1}{\left(1+\frac{r}{m}\right)^{n-k}}\right]$$

$$P_k = \frac{p}{\left(1+\frac{r}{m}\right)^{n-k+1}}$$

$$I_k = p\left[1 - \frac{1}{\left(1+\frac{r}{m}\right)^{n-k+1}}\right]$$