Technical Mathematics IV Homework

Instructions

1. Each assignment is to be done on one or more pieces of regular-sized notebook paper.
2. Your name and the assignment number should appear at the top of the first sheet.
3. Please do not use a red pen or red pencil to do the homework.
4. Please do not circle answers. Answer to a problem is not a single expression or number, it is the entire solution.
5. All relevant work is **required**. Problems are graded on the quality and correctness of the presented work.
6. Work through the homework problems referring to your notes and the lesson notes when necessary.
7. Redo the homework problems before an exam without referring to any other materials. It is best to do this more than once.

1. Complex Fractions

Simplify each of the following expressions.

1. $\dfrac{x-1}{1-\dfrac{1}{x}}$

2. $\dfrac{1-\dfrac{1}{x^2}}{x^3+x^2}$

3. $\dfrac{\dfrac{2}{x+2}-1}{-2x}$

4. $\dfrac{\dfrac{x-2}{x^2+2x-3}}{\dfrac{x^2-4}{x^2+5x+6}}$

5. $\dfrac{\dfrac{1}{x+2}-\dfrac{1}{2}}{x}$

6. $\dfrac{\dfrac{2}{x}+1}{1-\dfrac{4}{x^2}}$

7. $1-\dfrac{1}{1-\dfrac{1}{x-1}}$

8. $\dfrac{\dfrac{x}{x^2-9}-\dfrac{1}{x+3}}{\dfrac{x}{x-3}-1}$

9. $\dfrac{\dfrac{1}{2(x+h)}-\dfrac{1}{2x}}{\dfrac{h}{x+h}}$

10. $\dfrac{\dfrac{1}{x+h+2}-\dfrac{1}{x+2}}{\dfrac{h}{x+h+2}}$

11. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}}$

2. Exponents and Radicals

1. Write each of the following radicals in simplest radical form.

A. $\sqrt{169}$

B. $\sqrt{125}$

C. $\sqrt{98}$

D. $\sqrt{315}$

E. $\sqrt{192}$

F. $\sqrt{288}$

G. $\sqrt[3]{-135}$

H. $\sqrt[4]{32}$

2. Simplify each of the following expressions. Write each answer as a reduced rational number or, if possible, an integer.

A. 14^{-3}

B. -5^{-4}

C. $\dfrac{3}{6^{-2}}$

D. $\dfrac{3^{-3}}{2^{-4}}$

E. $\left(\dfrac{2}{5}\right)^{-3}$

3. Simplify each of the following expressions. Write the final answers without using negative exponents.

A. $\left(3x^{-2}y^{6}\right)\left(5^{-2}x^{2}y^{-7}\right)$

B. $\dfrac{12x^{3}y^{-7}}{9x^{-4}y^{-4}}$

C. $\dfrac{6x^{-5}y^{-3}y^{4}}{2x^{5}x^{2}y^{-6}}$

D. $\left(4x^{3}y^{-5}x^{4}y^{-3}\right)^{-2}$

E. $\left(\dfrac{6x^{-3}y^3z^2}{8x^{-2}z^{-4}}\right)^{-3}$

4. Simplify each of the following expressions. Write each answer as a reduced rational number or, if possible, an integer.

A. $64^{5/6}$

B. $(32)^{-6/5}$

C. $-25^{3/2}$

D. $(-27)^{-2/3}$

E. $\left(\dfrac{1}{32}\right)^{-4/5}$

5. Rewrite each of the following expressions in exponential form.

$$\sqrt[4]{x^5} \qquad \sqrt{x^4} \qquad \left(\sqrt[5]{x}\right)^3 \qquad \left(\sqrt[3]{x^2}\right)^4$$

6. Write each of the following expressions in two different ways using radicals.

$$x^{3/4} \qquad x^{5/2} \qquad x^{-2/3}$$

7. Simplify each of the following expressions. Write the final answers without using negative exponents.

A. $\left(\dfrac{x^{-1/2}}{x^{-1/3}}\right)^{-1/5}$

B. $\dfrac{x^{4/3}\,y^{1/10}}{x^{1/6}\,y^{2/5}}$

C. $\dfrac{\left(8\,x^{1/2}y^3\right)^{1/3}}{2\,x^{3/4}y^{1/3}}$

3. Functions

1. If $f(x) = x^2 - 3x + 5$, then find each of the following. Function notation must be used correctly.

A. $f(0)$

B. $f(-1)$

C. $f(3)$

D. $f(-2)$

E. $f\left(\frac{2}{3}\right)$

F. $f\left(-\frac{8}{7}\right)$

2. If $g(x) = \frac{2x}{3-x}$, then find each of the following. Function notation must be used correctly.

A. $g(0)$

B. $g(-4)$

C. $g(2)$

D. $g(-1)$

E. $g\left(-\frac{1}{2}\right)$

3. If $h(x) = 3x-4$, then find each of the following. Function notation must be used correctly.

A. $h(-2t)$

B. $h(y)$

C. $h\left(n^2\right)$

D. $h(-x)$

E. $h\left(3x-4\right)$

F. $h\left(2x^3+1\right)$

4. If $f(x) = -x^2 + 2x - 5$, then find each of the following. Function notation must be used correctly.

A. $f(x+1)$

B. $f(x-2)$

C. $f(2x-3)$

D. $f(4-x)$

4. Interval Notation

1. Write each of the following inequalities using interval notation.

A. $-2 \leq x \leq 5$

B. $3 < x < 8$

C. $x \geq -4$

D. $-7 \leq x < -2$

E. $x < 3$

F. $5 < x \leq 10$

G. $-9 < x \leq 9$

2. Write each interval as an inequality.

A. $(-\infty, 3]$

B. $[-6, -2)$

C. $(10, \infty)$

D. $(-1, 1]$

E. $[3, 5]$

F. $(-4, 0)$

3. Which of the following do not designate intervals?

A. $[3, -1]$

B. $(2, 4)$

C. $(1, -\infty)$

D. $(-2, \infty]$

E. $(1, 2]$

F. $(4, \infty)$

4. Classify each interval as either closed, open, or half-open/half-closed.

A. $[-2, -1]$

B. $[3, 8)$

C. $(-4, 10)$

D. $(0, 4]$

5. Write the domain of each function using interval notation.

A. $g(x) = 3x^2 - 2x + 1$

B. $h(x) = \dfrac{-4}{x - 2}$

C. $f(x) = \dfrac{x-2}{x^2+2x-3}$

D. $m(x) = \sqrt{x-5}$

E. $k(x) = \sqrt{3-2x}$

F. $s(x) = \dfrac{1}{\sqrt{x-1}}$

G. $w(x) = \dfrac{x+1}{x\sqrt{x+2}}$

5. Operations with Functions

In the following problems, compute and simplify each function or expression. Function notation must be used correctly in every problem.

$$f(x) = 2x - 3 \qquad g(x) = 2x^2 + 5x - 12 \qquad k(x) = \frac{1}{x+4}$$

1. $(f+g)(x)$

2. $(g-f)(x)$

3. $(g \cdot k)(x)$

4. $\left(\dfrac{g}{f}\right)(x)$

5. $\left(\dfrac{f}{k}\right)(x)$

6. $(f \circ g)(0)$

7. $(f \circ g)(x)$

8. $(g \circ f)(-1)$

9. $(g \circ f)(x)$

10. $(f \circ k)(2)$

11. $(f \circ k)(x)$

12. $(k \circ f)(0)$

13. $(k \circ f)(x)$

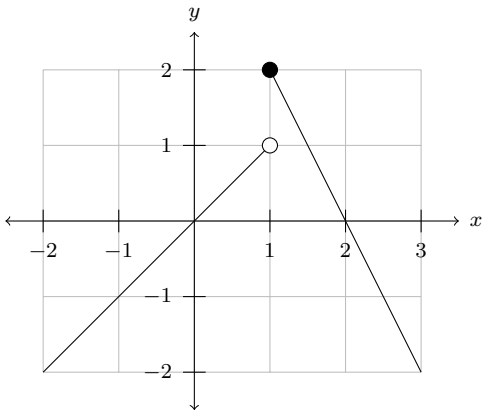
14. $\dfrac{f(x+h)-f(x)}{h}$

15. $\dfrac{g(x+h)-g(x)}{h}$

16. $\dfrac{k(x+h)-k(x)}{h}$

6. Graphs of Functions

1. The following is the graph of $y = f(x)$.



Find each of the following function values. Use function notation when writing your answers.

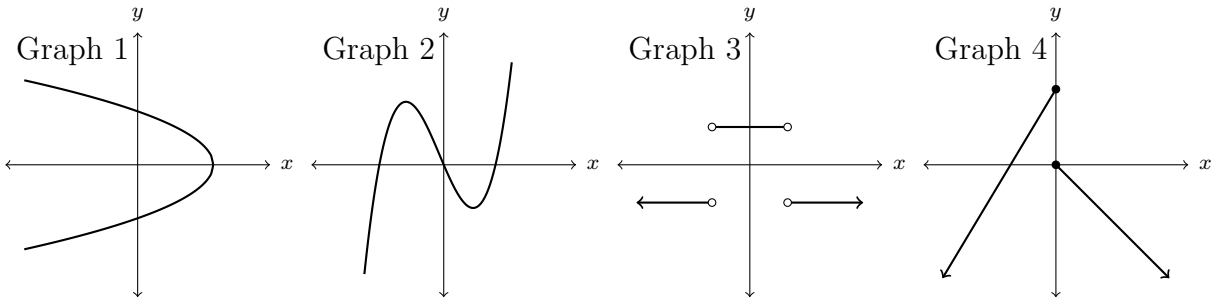
$f(3)$ $f(2)$ $f(1)$ $f(0)$ $f(-1)$ $f(-2)$

2. Graph $f(x) = 6x - 1$ when $-3 \le x \le 3$. Plot at least 7 points on the graph before drawing it.

3. Graph $f(x) = -2x^2 + 4x + 2$. Plot at least 9 points on the graph before drawing it.

4. Graph $f(x) = \sqrt{2x-1}$. Plot at least 9 points on the graph before drawing it.

5. Determine if each graph is the graph of a function.



6. Determine the range of each function. Write each range using interval notation.

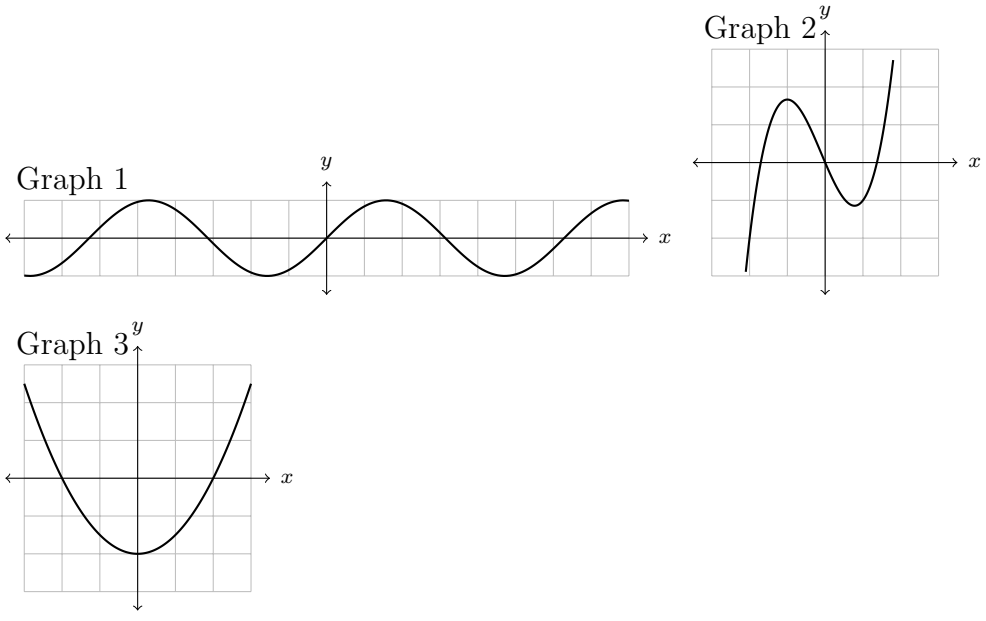
A. $f(x) = 4x - 3$

B. $g(x) = x^2 - 4x + 7$

C. $h(x) = -\sqrt{x+2}$

7. Even and Odd Functions

1. Classify each as the graph of an even function, odd function, or neither.



2. Determine if each function is even, odd, or neither.

A. $f(x) = 4x$

B. $g(x) = \dfrac{3}{x^2}$

C. $f(x) = -\dfrac{1}{x}$

D. $h(x) = 5x - 6$

E. $g(x) = 3x^4 + x^2$

F. $f(x) = \sqrt{x}$

8. Polynomial Functions

Complete each part for each polynomial function p listed below.

A. Find the zeros of $p(x)$.

B. Estimate the relative minimum points of the graph of p.

C. Estimate relative maximum points of the graph of p.

D. Determine the intervals on which the graph of p is increasing.

E. Determine the intervals on which the graph of p is decreasing.

F. Graph $y = p(x)$ including all zeros, relative minimums, and relative maximums.

G. Factor $p(x)$.

Problems

1. $p(x) = -x^3 - 5x^2 + 2x + 24$

2. $p(x) = x^3 - x^2 - 5x - 3$

3. $p(x) = x^4 - 18x^2 + 81$

4. $p(x) = x^5 - 3x^4 - 23x^3 + 51x^2 + 94x - 120$

9. Piecewise Polynomial Functions

1. Let function f be defined as follows.

$$f(x) = \begin{cases} 3x - 1 & \text{if} \quad x > 1 \\ x^2 - 4 & \text{if} \quad -1 < x \leq 1 \\ -x^3 & \text{if} \quad -2 < x \leq -1 \\ 5 - x & \text{if} \quad x < -2 \end{cases}$$

Find each of the following function values (if possible). Use function notation when writing your answers.

$f(3)$ $f(2)$ $f(1)$ $f(0)$ $f(-1)$ $f(-2)$ $f(-3)$ $f(-4)$

2. Sketch the graph of each function. Plot at least 7 points before drawing each graph.

A. $f(x) = \begin{cases} x + 1 & \text{if} \quad x \geq 0 \\ x^3 & \text{if} \quad x < 0 \end{cases}$

B. $f(x) = \begin{cases} x^2 + 2x & \text{if} \quad x < -1 \\ -x + 1 & \text{if} \quad -1 < x \leq 2 \\ -x^2 + 4x - 4 & \text{if} \quad x > 2 \end{cases}$

10. Rational Functions

Complete each part for each rational function r listed below.

A. Determine the domain of r.

B. Find the x-intercepts of the graph of r.

C. Find all vertical asymptotes of the graph of r.

D. Find any horizontal asymptotes of the graph of r.

E. Graph $y = r(x)$ including all x-intercepts and asymptotes.

Problems

1. $r(x) = \dfrac{2}{x - 1}$

2. $r(x) = \dfrac{4x - 5}{x + 2}$

3. $r(x) = \dfrac{1}{x^3 - 2x^2}$

4. $r(x) = \dfrac{4x^2 - 8x - 21}{x^2 - 9}$

5. $r(x) = \dfrac{-2x^2}{x^2 - 2x - 8}$

11. Graphs of the Sine and Cosine Functions

For each of the following functions, determine the amplitude and period, and sketch at least two cycles of the graph.

1. $y = \sin\left(\dfrac{1}{4}x\right)$

2. $y = -\cos(2x)$

3. $y = 2\sin\left(\dfrac{1}{2}x\right)$

4. $y = -3\cos\left(\dfrac{\pi x}{2}\right)$

5. $y = -\dfrac{1}{2}\sin\left(\dfrac{3}{4}x\right)$

6. $y = 10\cos(20x)$

12. Phase Shift

For each of the following functions, determine the amplitude, period, and phase shift and sketch at least two cycles of the graph.

1. $y = \sin\left(x + \dfrac{\pi}{4}\right)$

2. $y = 3\cos\left(\dfrac{1}{2}x - \dfrac{\pi}{6}\right)$

3. $y = -\sin(2x - \pi)$

4. $y = \cos(8\pi x + 2\pi)$

13. Graphs of the Tangent, Secant, Cosecant, and Cotangent Functions

Sketch at least two cycles of the graph of each of the following functions.

1. $y = \tan\left(\frac{1}{2}x\right)$

2. $y = -\tan(\pi x)$

3. $y = \cot(2x)$

4. $y = \frac{1}{4}\sec(4x)$

5. $y = -\sec\left(\frac{1}{3}x\right)$

6. $y = 2\csc(5x)$

7. $y = -\frac{1}{2}\csc(2\pi x)$

14. Simple Harmonic Motion

1. A weight is attached to the end of a spring. The weight is pushed 3 cm above equilibrium and released. The frequency of the weight's motion is 2 oscillations/s. Find the equation of the motion of the weight and graph at least two cycles of the equation starting with $t = 0$.

2. A weight attached to the end of a spring is moving with a frequency of 1/3 oscillations/s, a total displacement of 20 cm, and initial displacement $y = 0$. Find the equation of the motion of the weight and graph at least two cycles of the equation starting with $t = 0$.

15. Trigonometric Identities

Prove that the following identities are true.

1. $\sin(\theta)\csc(\theta) = 1$

2. $\dfrac{\cot(\theta)}{\csc(\theta)} = \cos(\theta)$

3. $\cos(\theta) + \dfrac{\cot(\theta)}{\csc(\theta)} = 2\cos(\theta)$

4. $1 + \cot^2(\theta) = \csc^2(\theta)$

5. $\sec(\theta) - \sec(\theta)\sin^2(\theta) = \cos(\theta)$

6. $\cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$

7. $\sec^2(\theta)\csc^2(\theta) = \sec^2(\theta) + \csc^2(\theta)$

8. $\csc(\theta) + \cot(\theta) = \dfrac{1}{\csc(\theta) - \cot(\theta)}$

9. $\dfrac{\tan^4(\theta) - 1}{\sec^2(\theta)(\tan(\theta) - 1)} = \tan(\theta) + 1$

10. $\cot(\theta) + \tan(\theta) + \sec(\theta)\csc(\theta) = 2\sec(\theta)\csc(\theta)$

11. $\sec(\theta) + \tan(\theta) = \dfrac{\cos(\theta)}{1 - \sin(\theta)}$

16. Sum and Difference Formulas

1. Prove the identity $\sin(x + \pi) = -\sin(x)$.

2. Prove the identity $\cos(x - 2\pi) = \cos(x)$.

3. Prove the identity $\sin\left(x - \dfrac{\pi}{2}\right) = -\cos(x)$.

4. Prove the identity $\tan(x + \pi) = \tan(x)$.

5. Simplify the expression $\sin(2x)\cos(3x) + \cos(2x)\sin(3x)$.

6. Simplify the expression $\dfrac{\tan(5x) - \tan(4x)}{1 + \tan(5x)\tan(4x)}$.

17. Double and Half Angle Formulas

1. Find $\cos(2\theta)$ if the terminal side of θ is in the first quadrant and $\cos(\theta) = \dfrac{4}{5}$.

2. Find $\tan(2\theta)$ if the terminal side of θ is in the second quadrant and $\sin(\theta) = \dfrac{3}{4}$.

3. Find $\sin(2\theta)$ if the terminal side of θ is in the fourth quadrant and $\tan(\theta) = -2$.

4. Suppose the terminal sides of θ and $\dfrac{\theta}{2}$ are in the first quadrant and $\sin(\theta) = \dfrac{3\sqrt{7}}{8}$. Find $\cos\left(\dfrac{\theta}{2}\right)$.

5. Suppose the terminal side of θ is in the fourth quadrant, the terminal side of $\dfrac{\theta}{2}$ is in the fourth quadrant, and $\tan(\theta) = -\sqrt{3}$. Find $\sin\left(\dfrac{\theta}{2}\right)$.

6. Use the facts that $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$ and $\sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$ to find the exact value of $\tan\left(\dfrac{\pi}{12}\right)$.

18. Inverse Trigonometric Functions

1. Suppose ABC is a right triangle with right angle C, $a = 6.2$, and $b = 7.4$. Determine angles A and B (in radians).

2. Suppose ABC is a right triangle with right angle C, $b = 4.0$, and $c = 9.1$. Determine angles A and B (in radians).

3. If $\cos(x) = -0.62$ and $\pi \leq x \leq 2\pi$, what is x?

4. If $\csc(x) = -7.5$ and $-\pi/2 \leq x \leq \pi/2$, what is x?

5. If $\cot(x) = 0.42$ and $0 < x < \pi/2$, what is x?

6. Determine the exact value of $\tan\left(\cos^{-1}\left(-\dfrac{5}{13}\right)\right)$ without using a calculator.

7. Determine the exact value of $\sin\left(\tan^{-1}\left(-\dfrac{1}{3}\right)\right)$ without using a calculator.

8. Determine the exact value of $\sec\left(\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)\right)$ without using a calculator.

19. Polar and Exponential Forms

1. Write each complex number in polar form. Put r in simplest radical form and give θ to the nearest degree.

A. $5 + 5\sqrt{3}i$

B. $-7 + 7i$

C. $-2\sqrt{3} - 2i$

D. $3 - 4i$

2. Write each complex number in rectangular form. Use three significant digits for a and b.

A. 4.50 cis 202°

B. 13.2 cis 76.0°

C. 10.7 cis 145°

3. Write each complex number in exponential form. Use three significant digits for r and θ.

A. $3 + 5i$

B. $-6 - 9i$

C. $4 - 2i$

4. Write each complex number in polar and rectangular form. Use either exact values or round to three significant digits.

A. $4\,e^{\frac{\pi}{2}i}$

B. $3\,e^{4.21i}$

C. $6\,e^{i}$