Technical Mathematics IV

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1 Course Description

Course: 2030:154 Technical Mathematics IV
Credits: 3
Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test. Functions and their graphs, polynomial and rational functions, polynomial equations, graphs of trigonometric functions, trigonometric identities and equations, analytic geometry, complex numbers in polar form.

2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to use a graphing calculator to solve a wide range of mathematical problems;
2. a complete understanding of graphs of polynomial functions including the concepts of relative maxima and minima, and increasing or decreasing intervals;
3. a complete understanding of graphs of rational functions including the concepts of vertical and horizontal asymptotes;
4. the ability to use a graphing approach to find the roots of polynomial functions of higher degree;
5. a complete understanding of the graphs of trigonometric functions including the concepts of amplitude, frequency, and phase shift;
6. an understanding of simple harmonic motion applications;
7. the ability to reproduce and use the most important trigonometric identities;
8. an understanding of the inverse trigonometric functions;
9. the ability to use the trigonometric and exponential forms of complex numbers.

3 Course Outline

1. Essential algebra topics for calculus
   (a) Operations with algebraic expressions
   (b) Exponents and radicals
   (c) Difference quotients
2. Functions and their graphs
   (a) Definitions of a function, domain, range
   (b) Continuity
   (c) Increasing and decreasing functions
   (d) Relative maxima and minima
   (e) Symmetry, even and odd functions
   (f) Piecewise defined functions
   (g) Operations on functions, composition of functions
3. Polynomial and rational functions
   (a) Polynomial functions and their graphs
   (b) Real solutions of polynomial equations
   (c) Complex solutions of polynomial equations
   (d) Rational functions and their graphs
   (e) Vertical and horizontal asymptotes
4. Analytic geometry
   (a) The circle
   (b) The parabola (optional)
   (c) The ellipse (optional)
   (d) The hyperbola (optional)
   (e) Translation of axes
   (f) The general second-degree equation
5. Graphing the trigonometric functions
   (a) Graphing the sine and cosine functions
   (b) Phase shift
   (c) Graphing the other trigonometric functions
   (d) Graphing composite curves
   (e) Simple harmonic motion
6. Trigonometric formulas and identities
   (a) Basic trigonometric identities
   (b) Sum and difference formulas
   (c) Double- and half-angle formulas
   (d) Trigonometric equations
   (e) Inverse trigonometric relations
   (f) Inverse trigonometric functions
7. Complex numbers
   (a) Trigonometric and exponential forms of complex numbers
   (b) Multiplication and division of complex numbers
   (c) Power and roots

4 Textbook

Technical Mathematics. Dale Ewen, Joan S. Gary, and James E. Trefzger. Pearson, Second Edition, 2005.

Chapter 4: 4.2
Chapter 5: 5.7
Chapter 8: 8.1, 8.2, 8.3
Chapter 12: 12.1, 12.2, 12.3, 12.4, 12.5
Chapter 13: 13.1, 13.2, 13.3, 13.4, 13.5
Chapter 14: 14.2, 14.3, 14.4
Chapter 16: 16.1, 16.2, 16.3
Chapter 20: 20.1, 20.2 (optional), 20.3 (optional), 20.4 (optional), 20.5, 20.6
Notes: Graphing trigonometric functions (Chapter 12) should be covered at the beginning of the course. Difference quotients, piecewise defined functions, operations on functions, composition of functions, rational functions, and vertical and horizontal asymptotes are not included in current textbook and should be covered using supplemental materials.

OpenStax Algebra and Trigonometry. OpenStax CNX, 2017. https://openstax.org/details/books/algebra-and-trigonometry.

5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

6 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

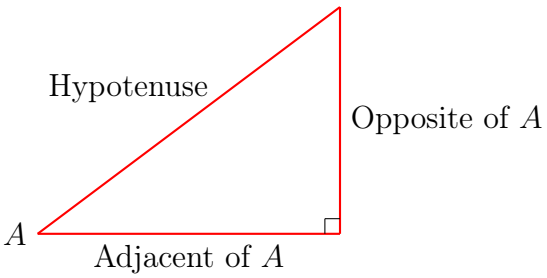
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

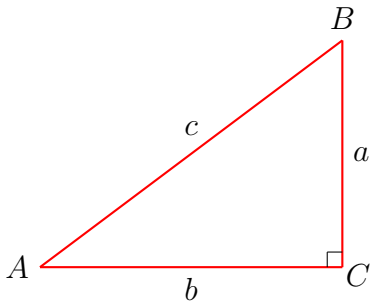
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



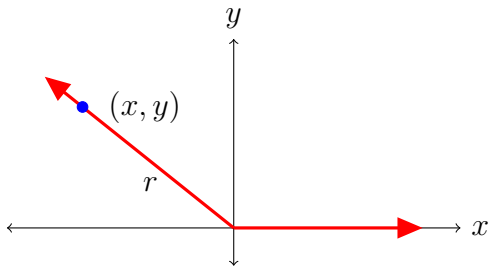
$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

$$\sin(A) = a/c \qquad \cos(A) = b/c \qquad \tan(A) = a/b$$
$$\csc(A) = c/a \qquad \sec(A) = c/b \qquad \cot(A) = b/a$$
$$\sin(B) = b/c \qquad \cos(B) = a/c \qquad \tan(B) = b/a$$
$$\csc(B) = c/b \qquad \sec(B) = c/a \qquad \cot(B) = a/b$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$
$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x, y) is a point on the terminal side of θ, and r is the distance from $(0, 0)$ to (x, y).



$$r^2 = x^2 + y^2$$

$$\sin(\theta) = y/r \qquad \cos(\theta) = x/r \qquad \tan(\theta) = y/x$$
$$\csc(\theta) = r/y \qquad \sec(\theta) = r/x \qquad \cot(\theta) = x/y$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of

the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \qquad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$