Technical Calculus I

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1 Course Description

Course: 2030:255 Technical Calculus I
Credits: 3
Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test. The derivative, applications of the derivative, derivatives of the trigonometric, logarithmic, and exponential functions, integration by antidifferentiation.

2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to calculate limits of functions;
2. the ability to identify the derivative as a particular kind of limit;
3. the ability to find the derivative of a function;
4. the ability to sketch the graph of a real-valued function by using derivatives;
5. the ability to recognize and solve technical problems by using differential calculus;
6. the ability to find the integral of a function.

3 Critical Thinking and Complex Reasoning Skills Learning Outcomes

Students will demonstrate foundational competency in creating and evaluating reasoned arguments and employing quantitative, qualitative, and normative information in such arguments.

Students create reasoned arguments and evaluate the reasonableness of arguments. They

1a. State the nature of controversies as propositions, including fact (i.e., what is), value (i.e., what should be), and policy (i.e., what steps can be taken) propositions;
1b. Recognize and choose the premises, purposes, audiences, and contexts of propositions;
1c. Recognize and choose the appropriate logic to support propositions, including symbolic, deductive, and inductive logic;
1d. (Not met explicitly in this course) Recognize and choose the appropriate information to support propositions, including the sources, authority, and biases of information;
1e. Recognize and be able to argue both sides of a proposition, and employ logic and information to challenge opposing propositions

Students employ the appropriate analysis and application of quantitative information, such that they:

2ai. Identify the value and limitations of magnitude (i.e., how large) and multitude (i.e., how many) measures;
2aii. Manipulate and express such measures with arithmetic, algebraic, geometric, and statistical methods;
2aiii. Manipulate and express such measures with graphs, charts, and tables;
2aiv. Manipulate and express such measures to solve practical and multistage problems.

In the course outline given below, a bold number indicates that the associated topic addresses the general education learning outcome with that number.

4 Course Outline

1. The derivative **1a, 1b, 1c, 1e**
   (a) Limits **2ai, 2aii, 2aiii**
   (b) Motion **2ai, 2aiv**
   (c) Tangent lines **2ai, 2aiv**
   (d) Definition of the derivative **2ai, 2aii, 2aiii, 2aiv**
   (e) Differentiation of polynomials **2aii**
   (f) The product and quotient rules **2aii**
   (g) The power rule **2aii, 2aiv**
   (h) Implicit differentiation **2aii, 2aiv**
   (i) Higher derivatives **2ai, 2aii**
2. Applications of the derivative **1a, 1b, 1c, 1e**

   (a) Curve sketching **2ai, 2aiii, 2aiv**
      - Relative extreme points
      - Concavity and inflection points
   (b) Optimization **2ai, 2aiv**
   (c) Related rates **2aiv**
   (d) Differentials **2aiv**
3. Derivatives of special functions **1a, 1b, 1c, 1e**
   (a) Trigonometric functions **2aii**
   (b) Inverse trigonometric functions **2aii**
   (c) Exponential functions **2aii**
   (d) Logarithmic functions **2aii**
   (e) Logarithmic differentiation **2aii, 2aiv**
4. Integration **1a, 1b, 1c, 1e**
   (a) Indefinite integrals **2ai, 2aii**
   (b) Applications of indefinite integrals **2aiv**
   (c) Definite integrals **2aii**
   (d) Area under a curve **2ai, 2aiv**
   (e) Integration by substitution **2aiv**
   (f) Exponential and logarithmic forms **2aii**
   (g) Trigonometric forms **2aii**
   (h) Inverse trigonometric forms **2aii**

5 Textbook

Technical Calculus with Analytic Geometry. Peter Kuhfittig. Brooks/Cole, Cengage Learning, Fifth Edition, 2013.

Chapter 1: 1.4
Chapter 2: 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9
Chapter 3: 3.1, 3.2, 3.4, 3.5, 3.6
Chapter 4: 4.1, 4.2, 4.3, 4.4, 4.5, 4.8
Chapter 6: 6.1 (optional), 6.2, 6.3, 6.4 (optional), 6.5, 6.6 (optional), 6.7, 6.8, 6.10
Chapter 7: 7.1, 7.2, 7.3, 7.4, 7.5

6 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

7 Artifact

During selected semesters, a student-produced artifact to be used for formative assessment of the effectiveness of the university's general education program will be collected, scanned, and stored securely. The artifact is an assignment covering limits, the definition of a derivative, the relationship between the derivative and the slope of a tangent line, the important rules for computing derivatives, velocity and acceleration, relative extrema and concavity, graphing, optimization, and related rates.

8 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a-b)(a+b)$$
$$x^2 + (a+b)x + ab = (x+a)(x+b)$$
$$acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

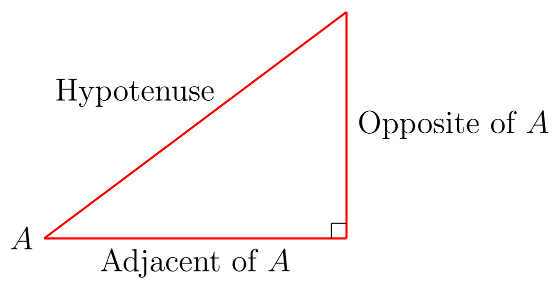
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

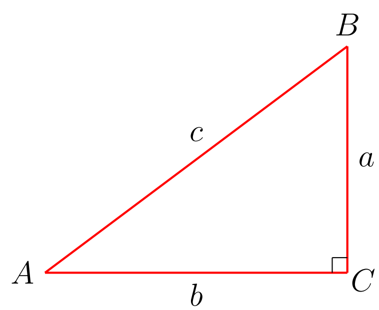
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

$$\sin(A) = a/c \qquad \cos(A) = b/c \qquad \tan(A) = a/b$$

$$\csc(A) = c/a \qquad \sec(A) = c/b \qquad \cot(A) = b/a$$

$$\sin(B) = b/c \qquad \cos(B) = a/c \qquad \tan(B) = b/a$$

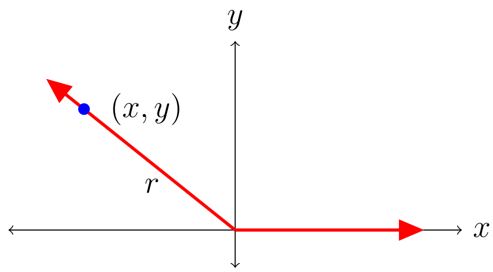
$$\csc(B) = c/b \qquad \sec(B) = c/a \qquad \cot(B) = a/b$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin ($(0,0)$). Point (x,y) is a point on the terminal side of θ, and r is the distance from $(0,0)$ to (x,y).



$$r^2 = x^2 + y^2$$

$$\sin(\theta) = y/r \qquad \csc(\theta) = r/y$$
$$\cos(\theta) = x/r \qquad \sec(\theta) = r/x$$
$$\tan(\theta) = y/x \qquad \cot(\theta) = x/y$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Differentiation and Integration Formulas

$$\frac{d}{dx}(u^n) = n\,u^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}(\sin(u)) = \cos(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos(u)) = -\sin(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan(u)) = \sec^2(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\sec(u)) = \sec(u)\,\tan(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\csc(u)) = -\csc(u)\,\cot(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cot(u)) = -\csc^2(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1}(u)) = -\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u}\,\frac{du}{dx} \qquad \frac{d}{dx}\left(e^u\right) = e^u\,\frac{du}{dx}$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \ (n \neq -1) \qquad \int \frac{1}{u} \, du = \ln |u| + C$$

$$\int e^u \, du = e^u + C \qquad \int \sin(u) \, du = -\cos(u) + C$$

$$\int \cos(u) \, du = \sin(u) + C \qquad \int \sec^2(u) \, du = \tan(u) + C$$

$$\int \csc^2(u) \, du = -\cot(u) + C \qquad \int \sec(u) \ \tan(u) \, du = \sec(u) + C$$

$$\int \csc(u) \ \cot(u) \, du = -\csc(u) + C \qquad \int \tan(u) \, du = -\ln |\cos(u)| + C$$

$$\int \cot(u) \, du = \ln |\sin(u)| + C \qquad \int \sec(u) \, du = \ln |\sec(u) + \tan(u)| + C$$

$$\int \csc(u) \, du = \ln |\csc(u) - \cot(u)| + C$$