Technical Calculus II

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1 Course Description

Course: 2030:356 Technical Calculus II
Credits: 3
Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test. Methods and applications of integration, first and second order differential equations, series expansion, Laplace transforms, partial derivatives, and double integrals.

2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to find the integral of a function by using partial fractions, integration by parts, or trigonometric substitution;
2. the ability to find areas and volumes by integration;
3. the ability to find the solutions of first-order differential equations using separation of variables or integrating factors;
4. the ability to solve second-order differential equations using standard methods and Laplace transforms;
5. the ability to use differential equations when solving real-world problems;
6. an understanding of the properties of numerical series and series of functions.

3 Course Outline

1. Applications of integration

   (a) Area between curves
   (b) Volumes of revolution—disk method
   (c) Volumes of revolution—shell method
   (d) Center of mass
2. Methods of integration
   (a) Partial fractions
   (b) Integration by parts
   (c) Trigonometric substitution
   (d) Integration using tables
   (e) Numerical methods
3. First-order differential equations
   (a) Solutions of differential equations
   (b) Separation of variables
   (c) Integrating factors
   (d) First-order linear equations
   (e) Applications
4. Second-order differential equations
   (a) Linear homogeneous case
   (b) Linear nonhomogeneous case
   (c) Applications
   (d) Laplace transforms
   (e) Using Laplace transforms to solve differential equations
5. Series
   (a) Convergence
   (b) Convergence tests
   (c) Power series
   (d) Maclaurin and Taylor series
   (e) Approximating series
   (f) Fourier series

4 Textbook

Technical Calculus with Analytic Geometry. Peter Kuhfittig. Brooks/Cole, Cengage Learning, Fifth Edition, 2013.

Chapter 4: 4.6, 4.7, 4.9
Chapter 5: 5.2, 5.3, 5.4, 5.5 (optional), 5.6 (optional)
Chapter 7: 7.6, 7.7, 7.8, 7.9
Chapter 11: 11.1, 11.2, 11.3, 11.4
  Note: Use supplemental material to cover exact differentials.
Chapter 12: 12.1, 12.2, 12.3, 12.4
Chapter 13: 13.1, 13.2, 13.3, 13.4
Chapter 10: 10.1, 10.2, 10.3, 10.4, 10.5, 10.6
  Note: There may not be enough time to cover chapter 10. If there is time, the most important sections are 10.3 and 10.6.

5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

6 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a-b)(a+b)$$
$$x^2 + (a+b)x + ab = (x+a)(x+b)$$
$$acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

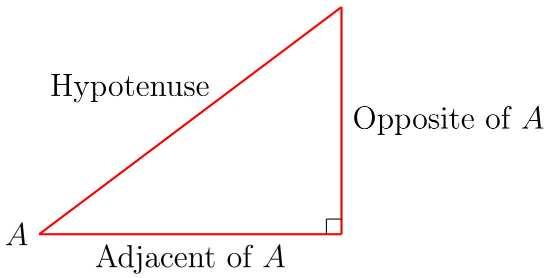
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

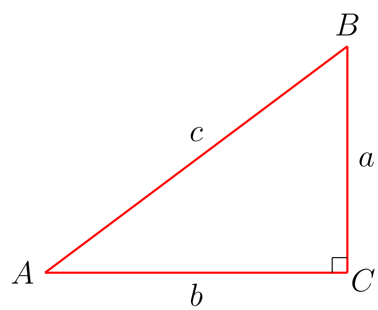
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

$$\sin(A) = a/c \qquad \cos(A) = b/c \qquad \tan(A) = a/b$$

$$\csc(A) = c/a \qquad \sec(A) = c/b \qquad \cot(A) = b/a$$

$$\sin(B) = b/c \qquad \cos(B) = a/c \qquad \tan(B) = b/a$$

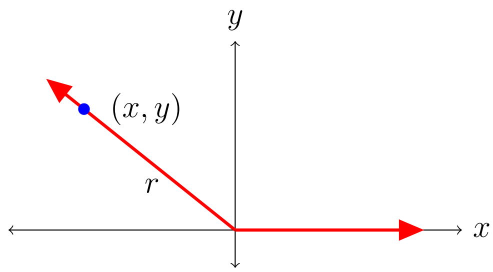
$$\csc(B) = c/b \qquad \sec(B) = c/a \qquad \cot(B) = a/b$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin ($(0,0)$). Point (x,y) is a point on the terminal side of θ, and r is the distance from $(0,0)$ to (x,y).



$$r^2 = x^2 + y^2$$

$$\begin{aligned} \sin(\theta) &= y/r & \csc(\theta) &= r/y \\ \cos(\theta) &= x/r & \sec(\theta) &= r/x \\ \tan(\theta) &= y/x & \cot(\theta) &= x/y \end{aligned}$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

Differentiation and Integration Formulas

$$\frac{d}{dx}(u^n) = n\,u^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}(\sin(u)) = \cos(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos(u)) = -\sin(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan(u)) = \sec^2(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\sec(u)) = \sec(u)\ \tan(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\csc(u)) = -\csc(u)\ \cot(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cot(u)) = -\csc^2(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1}(u)) = -\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u}\,\frac{du}{dx} \qquad \frac{d}{dx}\left(e^u\right) = e^u\,\frac{du}{dx}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \ (n \neq -1)$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\ \tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\ \cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = -\ln|\cos(u)| + C$$

$$\int \cot(u)\,du = \ln|\sin(u)| + C$$

$$\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$$

$$\int \csc(u)\,du = \ln|\csc(u) - \cot(u)| + C$$