Applied Cryptography

Course Description

Course: MATH 361 Applied Cryptography
Credits: 3
Prerequisite: A grade of C or better in MATH 261 Applied Finite Mathematics.
Course Description: Symmetric cryptography, modular arithmetic, stream and block ciphers, random numbers, Advanced Encryption Standard, public-key cryptography, key exchange, digital signatures, hash functions, message authentication.

Course Objectives

After completing this course the student should have the following competencies:

1. an understanding of the basic concepts of symmetric cryptography including symmetric keys, cleartext, ciphertext, and simple encryption methods such as the replacement cipher;
2. an understanding of the basic concepts of cryptanalysis and the methods used to attack an encryption system;
3. the ability to do computations in a ring of integers modulo n and an understanding of ciphers that use such rings;
4. an understanding of simple stream ciphers;
5. an understanding of the different types of random number generators that are used in cryptography and the ability to use random number generators to create ciphers such as a one-time pad;
6. an understanding of the important modes of operation for block ciphers;
7. a basic understanding of Galois fields and the ability to do computations in $GF(p^n)$;
8. an understanding of the structure of the Advanced Encryption Standard (AES) and the ability to encrypt and decrypt messages using the AES;
9. an understanding of the principles and common applications of public-key cryptography, and the primary number theory used in public-key cryptography;
10. an understanding of the RSA cryptosystem, the mathematics used in the system, and the ability to encrypt and decrypt cleartext using the system;
11. an understanding of the Diffie-Hellman key exchange and its applications;
12. an understanding of the basic digital signature protocol and the ability to use the RSA signature scheme;
13. an understanding of the purpose, security requirements, and properties of hash functions and the ability to use common hash function algorithms;
14. an understanding of the properties of message authentication codes and the ability to use hash functions to build a message authentication code.

Course Outline

1. Basics of cryptography
2. Symmetric encryption
    - Replacement cipher
3. Basic cryptanalysis
4. Modular arithmetic
    - The ring of integers modulo n
5. Stream ciphers
6. Random numbers
    - Random number generators
    - The one-time pad
7. Encryption using block ciphers
    - Modes of operation
8. The Advanced Encryption Standard (AES)
    - Galois fields
    - Structure of the AES
    - AES decryption
9. Public-key cryptography
    - Principles
    - One-way functions
    - Applications: key establishment, nonrepudiation, identification, encryption
    - The Euclidean and extended Euclidean algorithms
    - Euler's ϕ function
    - Fermat's little theorem and Euler's theorem
10. The RSA cryptosystem
11. Key exchange
    - Diffie-Hellman key exchange
    - Basic group theory (cyclic groups and their subgroups) (optional)
    - The discrete logarithm problem (optional)
    - Security of Diffie-Hellman key exchange (optional)
12. Digital signatures
    - Basic digital signature protocol
    - The RSA signature scheme
13. Hash functions
    - The purpose of hash functions
    - Hash function security requirements and properties
    - Hash function algorithms
14. Message authentication
    - Properties of message authentication codes
    - Building a message authentication code from a hash function

Bibliography

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