Technical Mathematics Area
Course Outlines

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1 Mathematics for Allied Health

1.1 Course Description

Course: 2030:130 Mathematics for Allied Health
Credits: 3
Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better.
Bulletin Description: Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better. The real number system, systems of measurement, conversions, linear equations, factoring, quadratic equations, graphing, linear systems, organizing data, averages, standard deviation, the normal distribution.

1.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to use basic algebra;
2. the ability to use ratios, proportions, variation to solve real-world problems;
3. a complete understanding of percentages and their uses;
4. an understanding of measurement systems;
5. the ability to convert measurements from one form to another form of measurement;
6. the ability to solve systems of linear equations and to use them in applications;
7. an understanding of the algebra of polynomials up to quadratic equations;
8. the ability to compute and use basic statistics.

1.3 Course Outline

1. The Real numbers
   (a) Fractions, decimals, percentages
   (b) Negative numbers
   (c) Powers and roots
2. Systems of measurement
   (a) Metric and U.S. systems: length, area, volume, mass, capacity, temperature, time, pressure, velocity
   (b) Reductions and conversions
3. Basic algebra
   (a) Algebraic expressions and operations: symbols and terminology, addition and subtraction of polynomials, multiplication and division of polynomials
   (b) Linear equations: finding solutions, formulas, word problems, ratios and proportions, mixture problems, dilution problems, percentages of solution problems, direct and indirect variation

(c) Factoring: removing common factors, differences of squares, trinomials
(d) Quadratic equations: Solving by factoring, using the quadratic formula, applications

4. Graphs
   (a) The rectangular coordinate system
   (b) Graphs of linear equations in two variables
5. Systems of equations
   (a) Solving a 2×2 system graphically
   (b) Solving a 2×2 system algebraically
   (c) Mixture problems
6. Statistics
   (a) Organizing data: tables, pie charts, bar graphs, etc.
   (b) Mode, median, mean
   (c) Standard deviation
   (d) The normal distribution

1.4 Textbook

Mathematics for the Health Sciences: A Comprehensive Approach. Joel R. Helms. Delmar, Cengage Learning, 2010.

Supplement: *Technical Mathematics.* Dale Ewen, Joan S. Gary, and James E. Trefzger. Pearson, Second Edition, 2005.

The following sections of *Mathematics for the Health Sciences* should be covered in this course and in order they are listed. As the text does not contain all of the topics that need to be covered in the course, supplementary material from *Technical Mathematics* is also listed. Sections 5.1, 6.2, 6.3, 8.2, and 8.3 of *Mathematics for the Health Sciences* may be consulted for examples and background information.

Chapter 1: 1.1, 1.2, 1.3, 1.4, 1.5
Chapter 2: 2.1, 2.2
Supplement: Section 6.1 from *Technical Mathematics*
Chapter 2: 2.3, 2.4, 2.5, 2.6, 2.7, 2.8
Supplement: Sections 5.1, 5.2, 5.3, 7.1, and 7.3 from *Technical Mathematics*
Chapter 3: 3.1, 3.2, 3.3, 3.5
Chapter 4: 4.1, 4.2, 4.3, 4.4
Chapter 5: 5.2, 5.7
Chapter 6: 6.1, 6.4, 6.5, 6.6
Chapter 9: 9.1, 9.2, 9.3, 9.4
Chapter 10: 10.1, 10.2, 10.3, 10.4, 10.5

1.5 Calculator Policy

All students are **required** to have a **scientific** or graphing calculator with minimum functionality equivalent to that of the **Texas Instruments TI–30X IIS** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No

exceptions to this policy will be made by the instructor.

2 Mathematics for Modern Technology

2.1 Course Description

Course: 2030:161 Mathematics for Modern Technology
Credits: 4
Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better.
Bulletin Description: Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better. Lines, linear regression, sets, counting, basic probability, basic statistics, binomial and normal distributions, mathematics of finance, symbolic logic, arguments, logic circuits.

2.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to graph lines, find equations of lines, and use linear regression lines in applications;
2. an understanding of the mathematics of finance including simple interest, compound interest, annuities, present value, future value, the APR, and consumer loans;
3. an understanding of sets and basic counting techniques and their uses;
4. the ability to compute simple probabilities and odds;
5. the ability to compute and apply modes, medians, means, expected values and standard deviations;
6. an understanding of the binomial and normal distributions and their applications;
7. an understanding of basic symbolic logic, arguments, and logic circuits.

2.3 General Education Learning Outcomes

Students will demonstrate foundational competency in creating and evaluating reasoned arguments and employing quantitative, qualitative, and normative information in such arguments. In particular, students employ the appropriate analysis and application of quantitative information, such that they:

1. Identify the value and limitations of magnitude (i.e., how large) and multitude (i.e., how many) measures;
2. Manipulate and express such measures with arithmetic, algebraic, geometric, and statistical methods;
3. Manipulate and express such measures with graphs, charts, and tables;
4. Manipulate and express such measures to solve practical and multistage problems.

In the course outline given below, a bold number indicates that the associated topic addresses the general education learning outcome with that number.

2.4 Course Outline

1. Lines
   (a) The concept of slope **1**
   (b) Graphing a line using its slope **1**, **3**
   (c) The point-slope, slope-intercept and general forms of a line
   (d) Mathematical modeling using linear functions **1**, **3**, **4**
   (e) Systems of linear equations and their applications
   (f) Linear regression and its applications **3**, **4**
2. Mathematics of Finance
   (a) Simple and compound interest: compound amount, present value, effective rate **1**, **2**
   (b) Ordinary annuities: present value, future value **1**, **2**
   (c) Amortization of loans **2**, **3**, **4**
   (d) Sinking funds **2**
   (e) IRAs, discount points, variable-rate mortgages, interest-only mortgages, the add-on method (all optional)
   (f) Difference equations (optional)
   (g) Financial mathematics using difference equations (optional)
3. Sets and Counting **1**
   (a) Basic set theory
   (b) The inclusion-exclusion principle
   (c) The multiplication principle
   (d) Permutations
   (e) Combinations
4. Probability
   (a) Experiments, outcomes, sample spaces, events **1**
   (b) The definition of probability **1**, **3**
   (c) Basic probability rules
   (d) Computing probabilities **1**, **2**
5. Statistics
   (a) Random variables, probability distributions, histograms **3**
   (b) The mean, expected value, and odds **1**, **2**, **4**
   (c) The median and mode **1**, **2**
   (d) Variance and standard deviation **1**, **2**, **4**
   (e) The binomial distribution **1**, **2**
   (f) The normal distribution and applications **1**, **2**, **4**
6. Logic
   (a) Symbolic logic: statements, and, or, not, implication
   (b) Truth tables
   (c) Logical equivalence
   (d) Arguments, rules of inference
   (e) Logic circuits

2.5 Textbook

Finite Mathematics: For the Managerial, Life, and Social Sciences. Soo T. Tan. Cengage Learning, Eleventh Edition, 2012.

The following sections of the text should be covered in this course:

Chapter 1: 1.1 (review), 1.2, 1.3, 1.4, 1.5
Note: There is no need to cover the distance formula or the equation of a circle in section 1.1.
Chapter 6: 6.1, 6.2, 6.3, 6.4
Chapter 7: 7.1, 7.2, 7.3, 7.4
Chapter 8: 8.1, 8.2, 8.3, 8.4, 8.5, 8.6
Chapter 5: 5.1, 5.2, 5.3
Note: The text does not contain material on IRAs, discount points, variable-rate mortgages, interest-only mortgages, the add-on method, or difference equations.
Appendix A: A.1, A.2, A.3, A.4, A.5, A.6

2.6 Calculator Policy

All students are **required** to have a **scientific** or graphing calculator with minimum functionality equivalent to that of the **Texas Instruments TI–30X IIS** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

2.7 Artifacts

During selected semesters, a student-produced artifact to be used for formative assessment of the effectiveness of the university's general education program will be collected, scanned, and stored securely. The artifact is an exam covering counting, probability, and basic statistics.

3 Technical Mathematics I

3.1 Course Description

Course: 2030:151 Technical Mathematics I
Credits: 2
Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better.
Bulletin Description: Prerequisite: Placement test or completion of 2010:052, 054, 057, or 084 with a grade of C or better. Fundamental concepts and operations, functions, graphs, factoring and algebraic fractions, variation, and quadratic equations.

3.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to solve basic algebra problems;
2. an understanding of functions and their graphs;
3. the ability to use basic factoring techniques;
4. an understanding of fractions with variables;
5. the ability to solve systems of linear equations;
6. the ability to solve quadratic equations.

3.3 Course Outline

1. Basic algebra
   (a) Scientific notation
   (b) Algebraic expressions
   (c) Addition and subtraction of algebraic expressions
   (d) Exponents and radicals
   (e) Multiplication of algebraic expressions
   (f) Division of algebraic expressions
   (g) Linear equations
   (h) Formulas and their applications
2. Functions and graphs
   (a) Functions and function notation
   (b) Graphing functions and simple algebraic equations
3. Factoring
   (a) Special products: product of two binomials, square of a binomial
   (b) Factoring out a common factor
   (c) Factoring the difference of two squares
   (d) Factoring trinomials

4. Algebraic fractions
   (a) Equivalent fractions
   (b) Reducing fractions
   (c) Multiplication and division of fractions
   (d) Addition and subtraction of fractions
   (e) Complex fractions
   (f) Equations with fractions
5. Systems of linear equations
   (a) Solving a 2×2 linear system by graphing
   (b) Solving a 2×2 linear system by substitution
   (c) Solving a 2×2 linear system by elimination
6. Quadratic equations
   (a) Solving a quadratic equation by factoring
   (b) Solving a quadratic equation using the quadratic formula

3.4 Textbook

OpenStax Algebra and Trigonometry. OpenStax CNX, 2017. https://openstax.org/details/books/algebra-and-trigonometry.

Chapter 1: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6
Chapter 2: 2.1, 2.2, 2.3, 2.5
Chapter 3: 3.1, 3.2
Chapter 5: 5.4
Chapter 11: 11.1

3.5 Calculator Policy

All students are **required** to have a **scientific** or graphing calculator with minimum functionality equivalent to that of the **Texas Instruments TI–30X IIS** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

3.6 Formula Policy

Some of the formulas that students will know by heart at the end of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4 Technical Mathematics II

4.1 Course Description

Course: 2030:152 Technical Mathematics II
Credits: 2
Prerequisites: 2030:151 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:151 or equivalent with a grade of C− or better, or placement test. Variation, equations of lines, Cramer's rule, right triangle trigonometry, oblique triangles, radian measure, complex numbers.

4.2 Course Outcomes

After completing this course the student should have the following competencies:

1. a complete understanding of the properties and equations of lines;
2. an understanding of basic right-triangle trigonometry and trigonometric functions;
3. the ability to apply right-triangle trigonometry to real-world situations;
4. the ability to use the laws of sines and cosines properly;
5. the ability to use radian measure of angles in theoretical and real-world applications;
6. the ability to solve problems using ratios, proportions, and variation;
7. the ability to use determinants and Cramer's rule to solve systems of linear equations;
8. the ability to perform basic arithmetical operations using complex numbers.

4.3 General Education Learning Outcomes

Students will demonstrate foundational competency in creating and evaluating reasoned arguments and employing quantitative, qualitative, and normative information in such arguments. In particular, students employ the appropriate analysis and application of quantitative information, such that they:

1. Identify the value and limitations of magnitude (i.e., how large) and multitude (i.e., how many) measures;
2. Manipulate and express such measures with arithmetic, algebraic, geometric, and statistical methods;
3. Manipulate and express such measures with graphs, charts, and tables;
4. Manipulate and express such measures to solve practical and multistage problems.

In the course outline given below, a bold number indicates that the associated topic addresses the general education learning outcome with that number.

4.4 Course Outline

1. Lines
   (a) The concept of slope **1**, **2**
   (b) Graphing a line using its slope **1**, **3**
   (c) Finding the slope-intercept and general forms of a line
   (d) Finding the slope-intercept form given a general form
   (e) Horizontal and vertical lines
   (f) Parallel and perpendicular lines
   (g) Distance between two points **1**, **2**
   (h) Midpoint of a line segment
2. Right-triangle trigonometry
   (a) Angles, measuring angles using degrees **1**
   (b) Defining the six trigonometric ratios
   (c) Computing the value of a trigonometric ratio using a calculator **1**, **2**
   (d) Using inverse trigonometric functions to find acute angles **2**
   (e) Solving right triangles **2**, **4**
   (f) Applications of right-triangle trigonometry **4**
3. Trigonometric functions
   (a) Angles made by rotation, measuring angles in standard position, coterminal angles **1**
   (b) Defining the six trigonometric functions
   (c) Signs of the trigonometric functions **1**
   (d) Reference angles **1**
   (e) Using reference angles to find other angles **1**, **4**
4. Oblique triangles
   (a) Solving triangles using the law of sines **1**, **2**, **4**
   (b) Solving triangles using the law of cosines **2**, **4**
5. Radian measure
   (a) Radian measure of angles **1**, **2**
   (b) Applications of radian measure **4**
6. Variation
   (a) Ratios, proportions and their applications **1**, **2**
   (b) Direct variation, inverse variation, applications of variation **2**, **4**
7. Cramer's Rule
   (a) 2×2 and 3×3 determinants
   (b) Solving 2×2 and 3×3 linear systems using Cramer's rule **1**, **4**
8. Complex numbers
   (a) Defining i **1**
   (b) Powers of i
   (c) The rectangular form of complex numbers
   (d) Addition and subtraction of complex numbers **2**
   (e) Multiplication of complex numbers **2**
   (f) Conjugates **2**

(g) Division of complex numbers **2**
(h) Using complex numbers to solve quadratic equations **4**

4.5 Textbook

OpenStax Algebra and Trigonometry. OpenStax CNX, 2017. https://openstax.org/details/books/algebra-and-trigonometry.

Chapter 2: 2.1, 2.2, 2.4, 2.5 (Include complex solutions to quadratic equations in this section.)
Chapter 5: 5.8
Chapter 7: 7.1, 7.2, 7.3, 7.4
Chapter 10: 10.1, 10.2
Chapter 11: 11.8

4.6 Calculator Policy

All students are **required** to have a **scientific** or graphing calculator with minimum functionality equivalent to that of the **Texas Instruments TI–30X IIS** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

4.7 Artifact

During selected semesters, a student-produced artifact to be used for formative assessment of the effectiveness of the university's general education program will be collected, scanned, and stored securely. The artifact is the comprehensive final exam given at the end of the course.

4.8 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$
$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

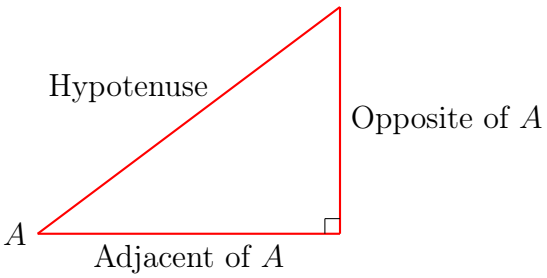
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

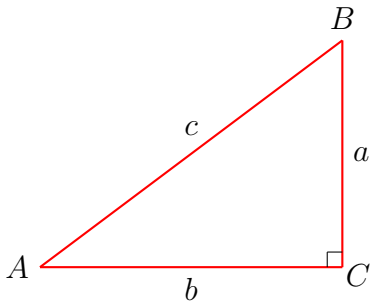
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

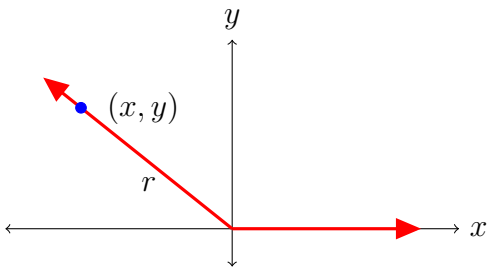
$$\begin{array}{lll} \sin(A) = a/c & \cos(A) = b/c & \tan(A) = a/b \\ \csc(A) = c/a & \sec(A) = c/b & \cot(A) = b/a \\ \sin(B) = b/c & \cos(B) = a/c & \tan(B) = b/a \\ \csc(B) = c/b & \sec(B) = c/a & \cot(B) = a/b \end{array}$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x, y) is a point on the terminal side of θ, and r is the distance from $(0, 0)$ to (x, y).



$$r^2 = x^2 + y^2$$

$$\begin{array}{ll} \sin(\theta) = y/r & \csc(\theta) = r/y \\ \cos(\theta) = x/r & \sec(\theta) = r/x \\ \tan(\theta) = y/x & \cot(\theta) = x/y \end{array}$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

5 Technical Mathematics III

5.1 Course Description

Course: 2030:153 Technical Mathematics III
Credits: 2
Prerequisites: 2030:152 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:152 or equivalent with a grade of C− or better, or placement test. Factoring, algebraic fractions, exponents and radicals, equations with radicals, equations in quadratic form, functions, their properties and graphs, exponential and logarithmic functions.

5.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to use basic factoring techniques;
2. an understanding of fractions with variables;
3. the ability to work with integral and fractional exponents;
4. the ability to solve equations with radicals or in quadratic form;
5. an understanding of functions including the definition of a function, function notation, evaluation of functions, and the concepts of domain and range;
6. an understanding of the relationship between a function and its graph;
7. the ability to perform some operations on functions (addition, subtraction, multiplication, division);
8. a complete understanding of exponential and logarithmic functions and their graphs.

5.3 General Education Learning Outcomes

Students will demonstrate foundational competency in creating and evaluating reasoned arguments and employing quantitative, qualitative, and normative information in such arguments. In particular, students employ the appropriate analysis and application of quantitative information, such that they:

1. Identify the value and limitations of magnitude (i.e., how large) and multitude (i.e., how many) measures;
2. Manipulate and express such measures with arithmetic, algebraic, geometric, and statistical methods;
3. Manipulate and express such measures with graphs, charts, and tables;
4. Manipulate and express such measures to solve practical and multistage problems.

In the course outline given below, a bold number indicates that the associated topic addresses the general education learning outcome with that number.

5.4 Course Outline

1. Factoring **1**, **2**
   (a) Special products: product of two binomials, square of a binomial, cube of a binomial
   (b) Factoring out a common factor
   (c) Factoring the difference of two squares, sum and difference of two cubes
   (d) Factoring trinomials
   (e) Factoring by grouping
2. Algebraic fractions **4**
   (a) Equivalent fractions
   (b) Reducing fractions
   (c) Multiplication and division of algebraic fractions
   (d) Addition and subtraction of algebraic fractions
3. Exponents and radicals
   (a) Integral exponents
   (b) Fractional exponents
   (c) Equations with radicals
   (d) Equations in quadratic form including complex solutions to quadratic equations **1**, **4**
4. Functions
   (a) Definition of a function, function notation, types of functions
   (b) Evaluation of a function **2**
   (c) The domain and range of a function expressed using inequality and interval notations **1**, **2**
   (d) Graphs of functions **1**, **3**
   (e) The relationship between a function and its graph (obtaining information from or about the graph of a function) **3**
   (f) Operations on functions (addition, subtraction, multiplication, division) **2**, **4**
5. Exponentials and logarithms
   (a) The exponential function **1**, **3**
   (b) The logarithm **1**, **3**
   (c) Properties of logarithms
   (d) Common logarithms
   (e) Natural logarithms
   (f) Solving exponential equations **2**, **4**
   (g) Solving logarithmic equations **2**, **4**
6. (Optional) Progressions and the Binomial Theorem
   (a) Arithmetic progressions
   (b) Geometric progressions
   (c) The Binomial Theorem

5.5 Textbook

OpenStax Algebra and Trigonometry. OpenStax CNX, 2017. https://openstax.org/details/books/algebra-and-trigonometry.

Chapter 1: 1.2, 1.3, 1.4, 1.5, 1.6
Chapter 2: 2.1, 2.6
Chapter 3: 3.1, 3.2
Chapter 5: 5.3, 5.4
Chapter 6: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6
Chapter 13: 13.1 (optional), 13.2 (optional), 13.3 (optional), 13.6 (optional)
Supplemental material needed: Laws of exponents, fractional exponents, multiplication and division of polynomials, factoring, rational expressions

5.6 Calculator Policy

All students are **required** to have a **scientific** or graphing calculator with minimum functionality equivalent to that of the **Texas Instruments TI–30X IIS** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

5.7 Artifact

During selected semesters, a student-produced artifact to be used for formative assessment of the effectiveness of the university's general education program will be collected, scanned, and stored securely. The artifact is the comprehensive final exam given at the end of the course.

5.8 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

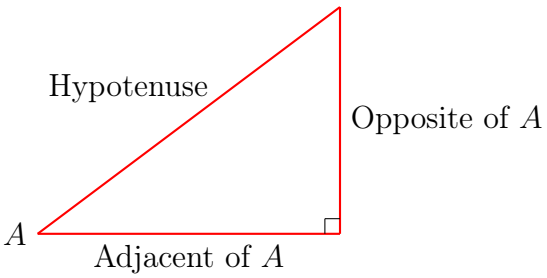
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

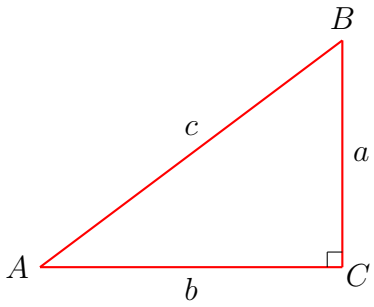
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

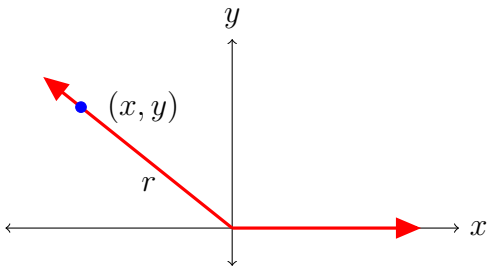
$$\begin{array}{lll} \sin(A) = a/c & \cos(A) = b/c & \tan(A) = a/b \\ \csc(A) = c/a & \sec(A) = c/b & \cot(A) = b/a \\ \sin(B) = b/c & \cos(B) = a/c & \tan(B) = b/a \\ \csc(B) = c/b & \sec(B) = c/a & \cot(B) = a/b \end{array}$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x, y) is a point on the terminal side of θ, and r is the distance from $(0, 0)$ to (x, y).



$$r^2 = x^2 + y^2$$

$$\begin{array}{ll} \sin(\theta) = y/r & \csc(\theta) = r/y \\ \cos(\theta) = x/r & \sec(\theta) = r/x \\ \tan(\theta) = y/x & \cot(\theta) = x/y \end{array}$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Factoring Formulas

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

6 Technical Mathematics IV

6.1 Course Description

Course: 2030:154 Technical Mathematics IV
Credits: 3
Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test. Functions and their graphs, polynomial and rational functions, polynomial equations, graphs of trigonometric functions, trigonometric identities and equations, analytic geometry, complex numbers in polar form.

6.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to use a graphing calculator to solve a wide range of mathematical problems;
2. a complete understanding of graphs of polynomial functions including the concepts of relative maxima and minima, and increasing or decreasing intervals;
3. a complete understanding of graphs of rational functions including the concepts of vertical and horizontal asymptotes;
4. the ability to use a graphing approach to find the roots of polynomial functions of higher degree;
5. a complete understanding of the graphs of trigonometric functions including the concepts of amplitude, frequency, and phase shift;
6. an understanding of simple harmonic motion applications;
7. the ability to reproduce and use the most important trigonometric identities;
8. an understanding of the inverse trigonometric functions;
9. the ability to use the trigonometric and exponential forms of complex numbers.

6.3 Course Outline

1. Essential algebra topics for calculus
   (a) Operations with algebraic expressions
   (b) Exponents and radicals
   (c) Difference quotients
2. Functions and their graphs
   (a) Definitions of a function, domain, range
   (b) Continuity
   (c) Increasing and decreasing functions
   (d) Relative maxima and minima
   (e) Symmetry, even and odd functions

(f) Piecewise defined functions
(g) Operations on functions, composition of functions

3. Polynomial and rational functions
   (a) Polynomial functions and their graphs
   (b) Real solutions of polynomial equations
   (c) Complex solutions of polynomial equations
   (d) Rational functions and their graphs
   (e) Vertical and horizontal asymptotes
4. Analytic geometry
   (a) The circle
   (b) The parabola (optional)
   (c) The ellipse (optional)
   (d) The hyperbola (optional)
   (e) Translation of axes
   (f) The general second-degree equation
5. Graphing the trigonometric functions
   (a) Graphing the sine and cosine functions
   (b) Phase shift
   (c) Graphing the other trigonometric functions
   (d) Graphing composite curves
   (e) Simple harmonic motion
6. Trigonometric formulas and identities
   (a) Basic trigonometric identities
   (b) Sum and difference formulas
   (c) Double- and half-angle formulas
   (d) Trigonometric equations
   (e) Inverse trigonometric relations
   (f) Inverse trigonometric functions
7. Complex numbers
   (a) Trigonometric and exponential forms of complex numbers
   (b) Multiplication and division of complex numbers
   (c) Power and roots

6.4 Textbook

OpenStax Algebra and Trigonometry. OpenStax CNX, 2017. https://openstax.org/details/books/algebra-and-trigonometry.

Chapter 1: 1.2 (only exponential rules), 1.4, 1.3, 1.6 (supplemental material is needed)
Chapter 2: 2.1, 2.6
Chapter 3: 3.1, 3.2, 3.4, 3.7
Chapter 5: 5.3, 5.5, 5.6, 5.7
Chapter 8: 8.1, 8.2, 8.3
Chapter 9: 9.1, 9.2, 9.3, 9.5
Chapter 10: 10.5 (supplemental material is needed)
Chapter 12: 12.1 (optional), 12.2 (optional), 12.3 (optional), 12.4

Notes: Graphing trigonometric functions (Chapter 8) should be covered at the beginning of the course. The equation of a circle and exponential form of a complex number are not included in current textbook and should be covered using supplemental materials. Laws of exponents and complex fractions are not covered in depth in this text so additional examples should be used.

6.5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

6.6 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

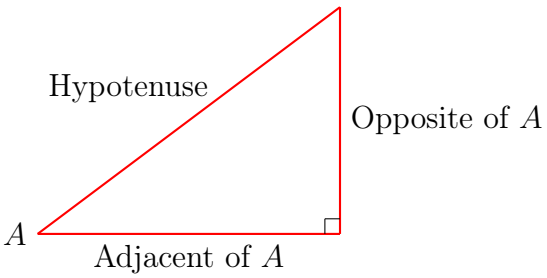
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

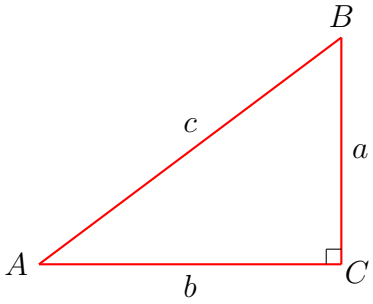
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

$$\sin(A) = a/c \qquad \cos(A) = b/c \qquad \tan(A) = a/b$$

$$\csc(A) = c/a \qquad \sec(A) = c/b \qquad \cot(A) = b/a$$

$$\sin(B) = b/c \qquad \cos(B) = a/c \qquad \tan(B) = b/a$$

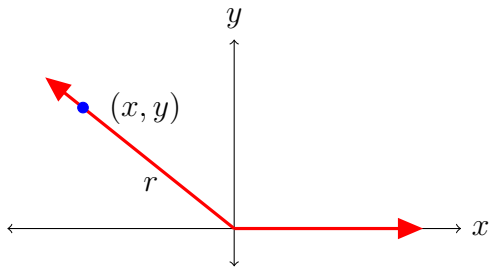
$$\csc(B) = c/b \qquad \sec(B) = c/a \qquad \cot(B) = a/b$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x,y) is a point on the terminal side of θ, and r is the distance from $(0,0)$ to (x,y).



$$r^2 = x^2 + y^2$$

$$\sin(\theta) = y/r \qquad \csc(\theta) = r/y$$
$$\cos(\theta) = x/r \qquad \sec(\theta) = r/x$$
$$\tan(\theta) = y/x \qquad \cot(\theta) = x/y$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \qquad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

7 Technical Calculus I

7.1 Course Description

Course: 2030:255 Technical Calculus I
Credits: 3
Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test. The derivative, applications of the derivative, derivatives of the trigonometric, logarithmic, and exponential functions, integration by antidifferentiation.

7.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to calculate limits of functions;
2. the ability to identify the derivative as a particular kind of limit;
3. the ability to find the derivative of a function;
4. the ability to sketch the graph of a real-valued function by using derivatives;
5. the ability to recognize and solve technical problems by using differential calculus;
6. the ability to find the integral of a function.

7.3 Critical Thinking and Complex Reasoning Skills Learning Outcomes

Students will demonstrate foundational competency in creating and evaluating reasoned arguments and employing quantitative, qualitative, and normative information in such arguments.

Students create reasoned arguments and evaluate the reasonableness of arguments. They

1a. State the nature of controversies as propositions, including fact (i.e., what is), value (i.e., what should be), and policy (i.e., what steps can be taken) propositions;
1b. Recognize and choose the premises, purposes, audiences, and contexts of propositions;
1c. Recognize and choose the appropriate logic to support propositions, including symbolic, deductive, and inductive logic;
1d. (Not met explicitly in this course) Recognize and choose the appropriate information to support propositions, including the sources, authority, and biases of information;
1e. Recognize and be able to argue both sides of a proposition, and employ logic and information to challenge opposing propositions

Students employ the appropriate analysis and application of quantitative information, such that they:

2ai. Identify the value and limitations of magnitude (i.e., how large) and multitude (i.e., how many) measures;
2aii. Manipulate and express such measures with arithmetic, algebraic, geometric, and statistical methods;
2aiii. Manipulate and express such measures with graphs, charts, and tables;
2aiv. Manipulate and express such measures to solve practical and multistage problems.

In the course outline given below, a bold number indicates that the associated topic addresses the general education learning outcome with that number.

7.4 Course Outline

1. The derivative **1a, 1b, 1c, 1e**
   (a) Limits **2ai, 2aii, 2aiii**
   (b) Motion **2ai, 2aiv**
   (c) Tangent lines **2ai, 2aiv**
   (d) Definition of the derivative **2ai, 2aii, 2aiii, 2aiv**
   (e) Differentiation of polynomials **2aii**
   (f) The product and quotient rules **2aii**
   (g) The power rule **2aii, 2aiv**
   (h) Implicit differentiation **2aii, 2aiv**
   (i) Higher derivatives **2ai, 2aii**
2. Applications of the derivative **1a, 1b, 1c, 1e**
   (a) Curve sketching **2ai, 2aiii, 2aiv**
      - Relative extreme points
      - Concavity and inflection points
   (b) Optimization **2ai, 2aiv**
   (c) Related rates **2aiv**
   (d) Differentials **2aiv**
3. Derivatives of special functions **1a, 1b, 1c, 1e**
   (a) Trigonometric functions **2aii**
   (b) Inverse trigonometric functions **2aii**
   (c) Exponential functions **2aii**
   (d) Logarithmic functions **2aii**
   (e) Logarithmic differentiation **2aii, 2aiv**
4. Integration **1a, 1b, 1c, 1e**
   (a) Indefinite integrals **2ai, 2aii**
   (b) Applications of indefinite integrals **2aiv**
   (c) Definite integrals **2aii**
   (d) Area under a curve **2ai, 2aiv**
   (e) Integration by substitution **2aiv**
   (f) Exponential and logarithmic forms **2aii**
   (g) Trigonometric forms **2aii**
   (h) Inverse trigonometric forms **2aii**

7.5 Textbook

Technical Calculus with Analytic Geometry. Peter Kuhfittig. Brooks/Cole, Cengage Learning, Fifth Edition, 2013.

Chapter 1: 1.4
Chapter 2: 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9
Chapter 3: 3.1, 3.2, 3.4, 3.5, 3.6
Chapter 4: 4.1, 4.2, 4.3, 4.4, 4.5, 4.8
Chapter 6: 6.1 (optional), 6.2, 6.3, 6.4 (optional), 6.5, 6.6 (optional), 6.7, 6.8, 6.10
Chapter 7: 7.1, 7.2, 7.3, 7.4, 7.5

7.6 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

7.7 Artifact

During selected semesters, a student-produced artifact to be used for formative assessment of the effectiveness of the university's general education program will be collected, scanned, and stored securely. The artifact is an assignment covering limits, the definition of a derivative, the relationship between the derivative and the slope of a tangent line, the important rules for computing derivatives, velocity and acceleration, relative extrema and concavity, graphing, optimization, and related rates.

7.8 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

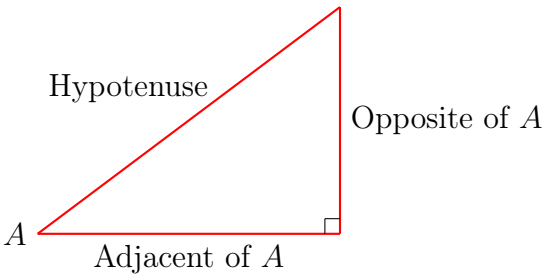
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

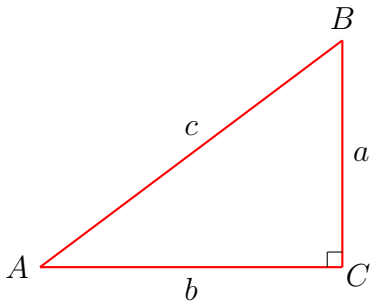
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

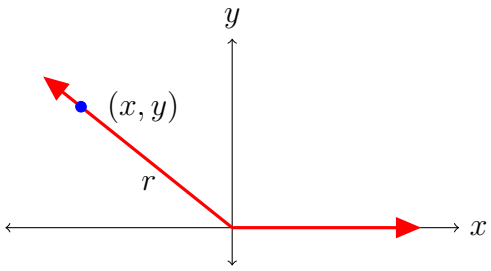
$$\begin{array}{lll} \sin(A) = a/c & \cos(A) = b/c & \tan(A) = a/b \\ \csc(A) = c/a & \sec(A) = c/b & \cot(A) = b/a \\ \sin(B) = b/c & \cos(B) = a/c & \tan(B) = b/a \\ \csc(B) = c/b & \sec(B) = c/a & \cot(B) = a/b \end{array}$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x, y) is a point on the terminal side of θ, and r is the distance from $(0, 0)$ to (x, y).



$$r^2 = x^2 + y^2$$

$$\begin{array}{ll} \sin(\theta) = y/r & \csc(\theta) = r/y \\ \cos(\theta) = x/r & \sec(\theta) = r/x \\ \tan(\theta) = y/x & \cot(\theta) = x/y \end{array}$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \qquad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Some of the formulas that students will know by heart at the end of this course are listed below.

Differentiation and Integration Formulas

$$\frac{d}{dx}(u^n) = n\,u^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}(\sin(u)) = \cos(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos(u)) = -\sin(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan(u)) = \sec^2(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\sec(u)) = \sec(u)\,\tan(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\csc(u)) = -\csc(u)\,\cot(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cot(u)) = -\csc^2(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1}(u)) = -\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u}\,\frac{du}{dx} \qquad \frac{d}{dx}\left(e^u\right) = e^u\,\frac{du}{dx}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C\ (n \neq -1) \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C \qquad \int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C \qquad \int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C \qquad \int \sec(u)\,\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\,\cot(u)\,du = -\csc(u) + C \qquad \int \tan(u)\,du = -\ln|\cos(u)| + C$$

$$\int \cot(u)\,du = \ln|\sin(u)| + C \qquad \int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$$

$$\int \csc(u)\,du = \ln|\csc(u) - \cot(u)| + C$$

8 Technical Calculus II

8.1 Course Description

Course: 2030:356 Technical Calculus II
Credits: 3
Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test. Methods and applications of integration, first and second order differential equations, series expansion, Laplace transforms, partial derivatives, and double integrals.

8.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to find the integral of a function by using partial fractions, integration by parts, or trigonometric substitution;
2. the ability to find areas and volumes by integration;
3. the ability to find the solutions of first-order differential equations using separation of variables or integrating factors;
4. the ability to solve second-order differential equations using standard methods and Laplace transforms;
5. the ability to use differential equations when solving real-world problems;
6. an understanding of the properties of numerical series and series of functions.

8.3 Course Outline

1. Applications of integration
   (a) Area between curves
   (b) Volumes of revolution—disk method
   (c) Volumes of revolution—shell method
   (d) Center of mass
2. Methods of integration
   (a) Partial fractions
   (b) Integration by parts
   (c) Trigonometric substitution
   (d) Integration using tables
   (e) Numerical methods
3. First-order differential equations
   (a) Solutions of differential equations
   (b) Separation of variables
   (c) Integrating factors

(d) First-order linear equations
(e) Applications

4. Second-order differential equations
   (a) Linear homogeneous case
   (b) Linear nonhomogeneous case
   (c) Applications
   (d) Laplace transforms
   (e) Using Laplace transforms to solve differential equations
5. Series
   (a) Convergence
   (b) Convergence tests
   (c) Power series
   (d) Maclaurin and Taylor series
   (e) Approximating series
   (f) Fourier series

8.4 Textbook

Technical Calculus with Analytic Geometry. Peter Kuhfittig. Brooks/Cole, Cengage Learning, Fifth Edition, 2013.

Chapter 4: 4.6, 4.7, 4.9
Chapter 5: 5.2, 5.3, 5.4, 5.5 (optional), 5.6 (optional)
Chapter 7: 7.6, 7.7, 7.8, 7.9
Chapter 11: 11.1, 11.2, 11.3, 11.4
Note: Use supplemental material to cover exact differentials.
Chapter 12: 12.1, 12.2, 12.3, 12.4
Chapter 13: 13.1, 13.2, 13.3, 13.4
Chapter 10: 10.1, 10.2, 10.3, 10.4, 10.5, 10.6
Note: There may not be enough time to cover chapter 10. If there is time, the most important sections are 10.3 and 10.6.

8.5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

8.6 Formula Policy

The formulas that students are required to know by heart at the beginning of this course are listed below.

Factoring Formulas

$$a^2 - b^2 = (a - b)(a + b)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$
$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Quadratic Formula

Let $ax^2 + bx + c = 0$ where a, b, and c are constants with $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations of Lines

Assume a line passes through (x_1, y_1) and (x_2, y_2) with slope m and y-intercept b.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = m\,(x - x_1) \qquad y = mx + b$$

Distance Formula

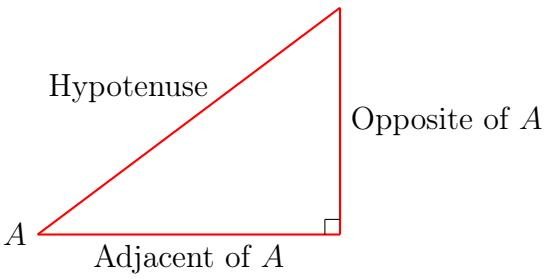
Let d be the distance between (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Parallel and Perpendicular Lines

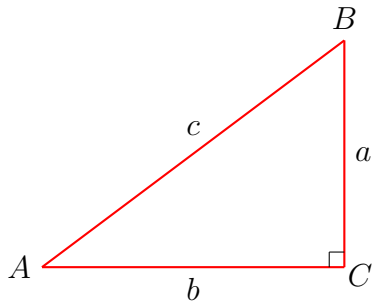
Suppose two lines have slopes m_1 and m_2 respectively. If the lines are parallel, then $m_1 = m_2$. If the lines are perpendicular, then $m_2 = -1/m_1$.

Right Triangle Trigonometry



$$\sin(A) = \frac{\text{Opposite of } A}{\text{Hypotenuse}} \qquad \cos(A) = \frac{\text{Adjacent of } A}{\text{Hypotenuse}} \qquad \tan(A) = \frac{\text{Opposite of } A}{\text{Adjacent of } A}$$

$$\csc(A) = \frac{\text{Hypotenuse}}{\text{Opposite of } A} \qquad \sec(A) = \frac{\text{Hypotenuse}}{\text{Adjacent of } A} \qquad \cot(A) = \frac{\text{Adjacent of } A}{\text{Opposite of } A}$$



$$a^2 + b^2 = c^2 \qquad A + B = 90^\circ$$

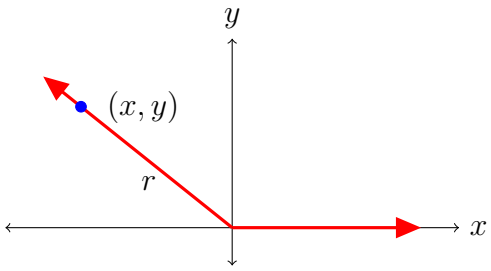
$$\begin{array}{lll} \sin(A) = a/c & \cos(A) = b/c & \tan(A) = a/b \\ \csc(A) = c/a & \sec(A) = c/b & \cot(A) = b/a \\ \sin(B) = b/c & \cos(B) = a/c & \tan(B) = b/a \\ \csc(B) = c/b & \sec(B) = c/a & \cot(B) = a/b \end{array}$$

$$A = \sin^{-1}(a/c) = \cos^{-1}(b/c) = \tan^{-1}(a/b)$$

$$B = \sin^{-1}(b/c) = \cos^{-1}(a/c) = \tan^{-1}(b/a)$$

General Trigonometry

Angle θ is shown below in standard position. The initial side of θ is the positive x-axis, and the vertex of θ is the origin $((0,0))$. Point (x, y) is a point on the terminal side of θ, and r is the distance from $(0,0)$ to (x, y).



$$r^2 = x^2 + y^2$$

$$\begin{array}{ll} \sin(\theta) = y/r & \csc(\theta) = r/y \\ \cos(\theta) = x/r & \sec(\theta) = r/x \\ \tan(\theta) = y/x & \cot(\theta) = x/y \end{array}$$

Radian Measure

$$180^\circ = \pi \text{ radians}$$

Let θ be the radian measure of a central angle of a circle with radius r. Let s be the length of the circular arc intercepted by θ, and A the area of the circular sector made by θ.

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Factoring Formulas

$$a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right) \qquad a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$$

Product Formulas

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

Exponents

$$a^{-n} = \frac{1}{a^n} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Logarithms

$$\log_b(mn) = \log_b(m) + \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n) \quad (m > 0,\ n > 0)$$

$$\log_b\left(m^n\right) = n\log_b(m) \quad (m > 0) \qquad \log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log(m) = \log_{10}(m) \qquad \ln(m) = \log_e(m) \qquad \log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$

Fundamental Trigonometric Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \qquad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Differentiation and Integration Formulas

$$\frac{d}{dx}(u^n) = n\,u^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}(\sin(u)) = \cos(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos(u)) = -\sin(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan(u)) = \sec^2(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\sec(u)) = \sec(u)\,\tan(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\csc(u)) = -\csc(u)\,\cot(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cot(u)) = -\csc^2(u)\,\frac{du}{dx} \qquad \frac{d}{dx}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1}(u)) = -\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx} \qquad \frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u}\,\frac{du}{dx} \qquad \frac{d}{dx}\left(e^u\right) = e^u\,\frac{du}{dx}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C\ (n \neq -1) \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C \qquad \int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C \qquad \int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C \qquad \int \sec(u)\,\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\,\cot(u)\,du = -\csc(u) + C \qquad \int \tan(u)\,du = -\ln|\cos(u)| + C$$

$$\int \cot(u)\,du = \ln|\sin(u)| + C \qquad \int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$$

$$\int \csc(u)\,du = \ln|\csc(u) - \cot(u)| + C$$

9 Advanced Trigonometry

9.1 Course Description

Course: 2030:260 Advanced Trigonometry
Credits: 2
Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:153 or equivalent with a grade of C− or better, or placement test. Horizontal circular curves, vertical curves, and spherical triangles, topics in astronomy.

9.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to identify a tangent line, secant line, diameter, radius, and chord of a circle;
2. the ability to find the length of an arc and a chord of a circle;
3. the ability to find the area of a sector, a segment, and between tangent lines and the circle;
4. the ability to determine the slope and external distance of a vertical curve;
5. the ability to identify the interior and dihedral angles in a spherical triangle;
6. an understanding of spherical coordinates;
7. the ability to use sine and cosine formulas for spherical triangles to solve theoretical and real-world applications;
8. an understanding of the PZS triangle and its applications.

9.3 Course Outline

1. Circles and circular curves
   (a) Arcs and central angles
   (b) Chords and segments
   (c) Secant and tangent lines
   (d) Perpendicular bisectors
   (e) Lengths of tangent lines, chords, curves, external distances and middle ordinates
   (f) Circular curve computation
2. Parabolic curves
   (a) Slope of a line (grade or gradient)
   (b) Distance of a line
   (c) Points of vertical curvature, intersection, and tangency
   (d) Tangent elevations
   (e) Basic form of a parabola
   (f) Finding the external distance of a vertical curve

3. Spherical trigonometry
   (a) Spherical triangles
   (b) Interior angles
   (c) Dihedral angles
   (d) Sine formulas for spherical triangles
   (e) Cosine formulas for sides of spherical triangles
   (f) Cosine formulas for angles of spherical triangles
4. Astronomy
   (a) Astronomical definitions
   (b) The PZS triangle
   (c) Applications of the PZS triangle
   (d) Sources of error in astronomical observations

9.4 Textbook

For circles, circular and parabolic curves use the following sections of the *Land Surveyor Reference Manual, Third Edition* by Harbin (2006).

Chapter 2: 2-31, 2-32, 2-33, 2-34, 2-35, 2-36, 2-37, 2-38, 2-39, 2-40, 2-41, 2-42, 2-43, 2-44, 2-45
Chapter 17: 17-1, 17-2, 17-3, 17-4, 17-5, 17-6, 17-7, 17-8, 17-9, 17-10, 17-14, 17-16
Chapter 22: 22-1, 22-2, 22-3, 22-4, 22-5, 22-6

For spherical trigonometry use the provided supplemental material or *Sphere, Spheroid and Projections for Surveyors* by Jackson (1987).

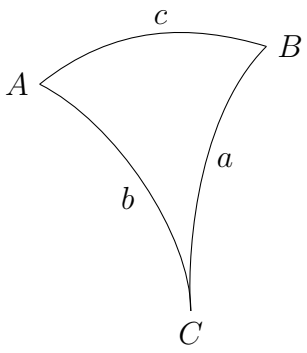
Chapter 2: 2.1, 2.2, 2.3, 2.4
Chapter 3: 3.1, 3.2

9.5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

9.6 Formula Policy

The formulas that students are required to know by heart at the end of this course are listed below.



$$\frac{\sin(a)}{\sin(A)} = \frac{\sin(b)}{\sin(B)} = \frac{\sin(c)}{\sin(C)}$$

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

10 Technical Data Analysis

10.1 Course Description

Course: 2030:345 Technical Data Analysis
Credits: 2
Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:154 or equivalent with a grade of C− or better, or placement test. Data summarization including graphing presentation, numerical measures, introduction to probability, confidence intervals, and hypothesis testing.

10.2 Course Outcomes

After completing this course the student should have the following competencies:

1. an understanding of the nature of data sets;
2. an understanding of the process of designing of a statistical study;
3. an understanding of the importance of using different methods of collecting data sets;
4. the ability to summarize data based on the methods of descriptive statistics: graphing presentation and numerical measures (measures of central tendency, measures of variation, measures of position);
5. the ability to use the counting techniques (multiplication principle, permutations, combinations);
6. the ability to compute probabilities, odds, and expected values;
7. an understanding of the normal distribution and its applications;
8. the ability to compute confidence intervals;
9. the ability to do hypothesis testing;
10. the ability to apply all of the above to real-life projects;
11. the ability to use technology such as the graphing calculator and spreadsheet software.

10.3 Course Outline

1. Introduction to statistics
   (a) The nature of data
   (b) Uses and abuses of statistics
   (c) Design of experiments
2. Describing, exploring, and comparing data
   (a) Summarizing data with frequency tables
   (b) Pictures of data
   (c) Measures of central tendency
   (d) Measures of variation
   (e) Measures of position

   (f) Exploratory data analysis
3. Correlation and regression
   (a) Linear regression
   (b) Correlation
4. Probability
   (a) Counting
   (b) Fundamentals
5. Probability distributions
   (a) Random variables
   (b) Binomial probability distributions
   (c) The normal distribution
   (d) The central limit theorem
   (e) Approximating a binomial distribution
   (f) Determining normality
6. Estimates and sample sizes
   (a) Estimating population means
   (b) Determining sample size
   (c) Estimating population proportions
7. Hypothesis testing

10.4 Textbook

Elementary Statistics. Mario F. Triola. Pearson.

10.5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

11 Advanced Topics in Technical Mathematics

11.1 Course Description

Course: 2030:480 Advanced Topics in Technical Mathematics
Credits: 2
Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test.
Bulletin Description: Prerequisites: 2030:255 or equivalent with a grade of C− or better, or placement test. Matrices, introduction to series, partial derivatives, least squares adjustments, and coordinate systems.

11.2 Course Outcomes

After completing this course the student should have the following competencies:

1. the ability to perform matrix addition, scalar multiplication of matrices, transposing of matrices, and matrix multiplication;
2. the ability to find the solution of a linear system of equations using matrices;
3. an understanding of power series;
4. the ability to find the partial derivative of a function;
5. the ability to recognize and solve technical problems by using partial derivatives;
6. the ability to calculate the least squares adjustment using the observed equation and matrix methods;
7. an understanding of polar, spherical, geocentric and geodetic coordinate systems and position computation.

11.3 Course Outline

1. Matrices
   (a) Basic operations on matrices
   (b) Multiplication of matrices
   (c) The inverse of a matrix
   (d) The transpose of a matrix
   (e) Solving a system of linear equations using matrices
2. Series
   (a) Introduction to series
   (b) Power series
3. Partial derivatives
   (a) Power rule
   (b) Product rule
   (c) Chain rule
   (d) Partial derivatives

(e) Applications of partial derivatives

4. Least squares adjustments
   (a) Introduction to least squares adjustments
   (b) Conditions for least squares
   (c) Observation equation method
   (d) Matrix methods in least squares adjustments
5. Coordinate systems
   (a) The polar coordinate system
   (b) The spherical coordinate system
   (c) The geocentric and geodetic coordinate systems
   (d) The local geodetic coordinate system
   (e) Ellipsoidal radii of curvature

11.4 Textbook

Supplemental material is used for this course.

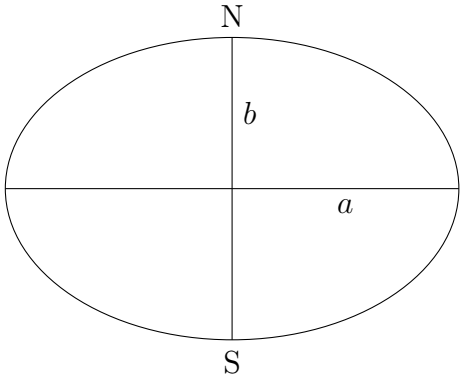
The course 2980:310, Survey Computations and Adjustments has Topics in Advanced Mathematics as a corequisite. In order to ensure that students taking these two courses during a single semester have learned the mathematics they need for Survey Computations and Adjustments, the order of the supplemental material should be followed in Advanced Topics in Technical Mathematics.

11.5 Calculator Policy

All students are **required** to have a **graphing** calculator with minimum functionality equivalent to that of the **Texas Instruments TI–83** calculator. Every student is **required** to have possession of their calculator by the end of the first week of classes. No exceptions to this policy will be made by the instructor.

11.6 Formula Policy

The formulas that students are required to know by heart at the end of this course are listed below.



a = semi-major axis

b = semi-minor axis

ϕ = latitude

$$f = \frac{a-b}{a}$$

$$e^2 = \frac{a^2-b^2}{a^2}$$

$$M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin(\phi)\right)^{3/2}}$$

$$N = \frac{a}{\sqrt{1-e^2\sin^2(\phi)}}$$

12 Applied Finite Mathematics

12.1 Course Description

Course: 2030:216 Applied Finite Mathematics
Credits: 3
Prerequisite: 2030:153 Technical Mathematics III with a grade of C- or better, or placement test.
Bulletin Description: Prerequisite: 2030:153 with a grade of C- or better, or placement test. Number systems, integer rings, finite fields, number theory algorithms, prime numbers and primality tests, factoring, and random numbers.

12.2 Course Objectives

After completing this course the student should have the following competencies:

1. an understanding of binary, octal, and hexadecimal numbers;
2. an understanding of integer rings and finite fields;
3. the ability to use the Euclidean algorithm, the Chinese remainder theorem, Euler's ϕ function, Fermat's little theorem, and Euler's theorem;
4. an understanding of the different methods that can be used to find prime numbers;
5. an understanding of factoring algorithms and their uses;
6. an understanding of the processes used to generate random numbers.

12.3 Course Outline

1. Number systems
   - Representations of numbers
   - Binary, octal, and hexadecimal numbers
2. Modular arithmetic
3. Integer rings
4. Finite fields
   - Galois fields
   - Extension fields
5. Euclidean and extended Euclidean algorithms
6. Chinese remainder theorem
7. Euler's ϕ function
8. Fermat's little theorem
9. Euler's theorem
10. Prime numbers
    - Finding prime numbers: Sieve of Erastothenes etc.
    - Primality tests

11. Factoring
    - Divisibility and unique factorization
    - Factoring algorithms
12. Random numbers
    - Random and psuedorandom number generators

12.4 Bibliography

1. Margaret Cozzens and Steven Miller. *The Mathematics of Encryption: An Elementary Introduction*. Mathematical World, V. 29. American Mathematical Society, 2013.
2. Samuel Wagstaff. *The Joy of Factoring*. Student Mathematical Library, V. 68. American Mathematical Society, 2013.
3. Christof Paar and Jan Pelzl. *Understanding Cryptography: A Textbook for Students and Practitioners*. Springer, 2010.
4. Eric Gossett. *Discrete Mathematics with Proof, Second Edition*. Wiley, 2009.
5. Jeffrey Hoffstien, Jill Pipher, and Joseph Silverman. *An Introduction to Mathematical Cryptography*. Springer, 2008.
6. Shafi Goldwasser and Mihir Bellare. *Lecture Notes on Cryptography*. Available at http://cseweb.ucsd.edu/~mihir/papers/gb.html, 2008.
7. Albrecht Beutelspacher. *Cryptology*. Mathematical Association of America, 1994.

13 Applied Cryptography

13.1 Course Description

Course: 2030:361 Applied Cryptography
Credits: 3
Prerequisite: A grade of C- or better in either 2030:154 Technical Mathematics IV or 2030:216 Applied Finite Mathematics
Bulletin Description: Prerequisite: A grade of C- or better in either 2030:154 or 2030:216. Symmetric cryptography, modular arithmetic, stream and block ciphers, random numbers, Advanced Encryption Standard, public-key cryptography, key exchange, digital signatures, hash functions, message authentication.

13.2 Course Objectives

After completing this course the student should have the following competencies:

1. an understanding of the basic concepts of symmetric cryptography including symmetric keys, cleartext, ciphertext, and simple encryption methods such as the replacement cipher;
2. an understanding of the basic concepts of cryptanalysis and the methods used to attack an encryption system;
3. the ability to do computations in a ring of integers modulo n and an understanding of ciphers that use such rings;
4. an understanding of simple stream ciphers;
5. an understanding of the different types of random number generators that are used in cryptography and the ability to use random number generators to create ciphers such as a one-time pad;
6. an understanding of the important modes of operation for block ciphers;
7. a basic understanding of Galois fields and the ability to do computations in $GF(p^n)$;
8. an understanding of the structure of the Advanced Encryption Standard (AES) and the ability to encrypt and decrypt messages using the AES;
9. an understanding of the principles and common applications of public-key cryptography, and the primary number theory used in public-key cryptography;
10. an understanding of the RSA cryptosystem, the mathematics used in the system, and the ability to encrypt and decrypt cleartext using the system;
11. an understanding of the Diffie-Hellman key exchange and its applications;
12. an understanding of the basic digital signature protocol and the ability to use the RSA signature scheme;
13. an understanding of the purpose, security requirements, and properties of hash functions and the ability to use common hash function algorithms;
14. an understanding of the properties of message authentication codes and the ability to use hash functions to build a message authentication code.

13.3 Course Outline

1. Basics of cryptography
2. Symmetric encryption
    - Replacement cipher
3. Basic cryptanalysis
4. Modular arithmetic
    - The ring of integers modulo n
5. Stream ciphers
6. Random numbers
    - Random number generators
    - The one-time pad
7. Encryption using block ciphers
    - Modes of operation
8. The Advanced Encryption Standard (AES)
    - Galois fields
    - Structure of the AES
    - AES decryption
9. Public-key cryptography
    - Principles
    - One-way functions
    - Applications: key establishment, nonrepudiation, identification, encryption
    - The Euclidean and extended Euclidean algorithms
    - Euler's ϕ function
    - Fermat's little theorem and Euler's theorem
10. The RSA cryptosystem
11. Key exchange
    - Diffie-Hellman key exchange
    - Basic group theory (cyclic groups and their subgroups) (optional)
    - The discrete logarithm problem (optional)
    - Security of Diffie-Hellman key exchange (optional)
12. Digital signatures
    - Basic digital signature protocol
    - The RSA signature scheme
13. Hash functions
    - The purpose of hash functions
    - Hash function security requirements and properties
    - Hash function algorithms
14. Message authentication
    - Properties of message authentication codes
    - Building a message authentication code from a hash function

13.4 Bibliography

1. Jonathan Katz and Yehuda Lindell. *Introduction to Modern Cryptography, 2nd Edition*, Chapman \ Hall/CRC Cryptography and Network Security Series. Chapman and Hall/CRC, 2014.
2. J Vacca. *Computer and Information Security Handbook, 2nd Edition*. Elsevier, 2013.
3. Margaret Cozzens and Steven Miller. *The Mathematics of Encryption: An Elementary Introduction*. Mathematical World, V. 29. American Mathematical Society, 2013.
4. Samuel Wagstaff. *The Joy of Factoring*. Student Mathematical Library, V. 68. American Mathematical Society, 2013.
5. Alasdair McAndrew. *Introduction to Cryptography with Open-Source Software*, Discrete Mathematics and its Applications series. CRC Press, 2011
6. Niels Ferguson, Bruce Schneier, Tadayoshi Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley, 2010.
7. Christof Paar and Jan Pelzl. *Understanding Cryptography: A Textbook for Students and Practitioners*. Springer, 2010.
8. Jeffrey Hoffstien, Jill Pipher, and Joseph Silverman. *An Introduction to Mathematical Cryptography*. Springer, 2008.
9. Shafi Goldwasser and Mihir Bellare. *Lecture Notes on Cryptography*. Available at http://cseweb.ucsd.edu/~mihir/papers/gb.html, 2008.
10. Niels Ferguson. *Practical Cryptography*. Wiley, 2003.
11. Bruce Schneier. *Applied Cryptography: Protocols, Algorithms, and Source Code in C*. John Wiley \ Sons, Second Edition, 1996.
12. Albrecht Beutelspacher. *Cryptology*. Mathematical Association of America, 1994.

14 Applied Cryptanalysis

14.1 Course Description

Course: 2030:461 Applied Cryptanalysis
Credits: 3
Prerequisite: 2030:361 Applied Cryptography with a grade of C- or better
Bulletin Description: Prerequisite: 2030:361 with a grade of C- or better. Cryptanalysis concepts; cryptanalysis of symmetric and public key cryptosystems, key exchange systems, and digital signatures; hash function collision resistance; cryptanalysis with quantum computers.

14.2 Course Objectives

After completing this course the student should have the following competencies:

1. an understanding of the basic concepts of cryptanalysis and the methods used to attack an encryption system;
2. the ability to implement an exhaustive key search against a symmetric cryptosystem;
3. an understanding of basic factoring algorithms and the ability to use those algorithms;
4. an understanding of how to attack the RSA cryptosystem using a factoring algorithm;
5. an understanding of how to find a brute force solution to the discrete logarithm problem, and the ability to conduct a man-in-the-middle attack against the Diffie-Hellman key exchange;
6. an understanding of how to attack the RSA signature scheme;
7. an understanding of the concept of collision resistance and The Birthday Attack;
8. an understanding of what the development of quantum computers will mean to the security of public key cryptography.

14.3 Course Outline

1. General mathematical cryptanalysis concepts
   - Key recovery vs. decryption
   - Kerckhoff's Principle
2. Cryptanalysis of symmetric cryptosystems
   - Symmetric cryptosystems
   - Brute force attacks
     - Exhaustive key search
     - Key lengths and security levels
3. Public key cryptography review
4. Review of the RSA cryptosystem
5. Factoring algorithms

6. Mathematical attacks on RSA
    - Preventing mathematical attacks
7. Key exchange
    - Review of the Diffie-Hellman key exchange
    - The discrete logarithm problem
        - Brute force solutions
    - The generalized Diffie-Hellman problem
    - Man-in-the-middle attack against the Diffie-Hellman key exchange
8. Digital signatures
    - Review of the principles of digital signatures
    - Review of the RSA signature scheme
    - Attacks against the RSA signature scheme
9. Hash functions
    - Collision resistance
    - The Birthday Attack
10. Implications of quantum computers on public key cryptography

14.4 Bibliography

1. Jonathan Katz and Yehuda Lindell. *Introduction to Modern Cryptography, 2nd Edition*, Chapman \ Hall/CRC Cryptography and Network Security Series. Chapman and Hall/CRC, 2014.
2. J Vacca. *Computer and Information Security Handbook, 2nd Edition*. Elsevier, 2013.
3. Margaret Cozzens and Steven Miller. *The Mathematics of Encryption: An Elementary Introduction*. Mathematical World, V. 29. American Mathematical Society, 2013.
4. Samuel Wagstaff. *The Joy of Factoring*. Student Mathematical Library, V. 68. American Mathematical Society, 2013.
5. Alasdair McAndrew. *Introduction to Cryptography with Open-Source Software*, Discrete Mathematics and its Applications series. CRC Press, 2011
6. Niels Ferguson, Bruce Schneier, Tadayoshi Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley, 2010.
7. Christof Paar and Jan Pelzl. *Understanding Cryptography: A Textbook for Students and Practitioners*. Springer, 2010.
8. M. Jason Hinek. *Cryptanalysis of RSA and Its Variants*, Cryptography and Network Security Series. Chapman & Hall/CRC, 2009.
9. Antoine Joux. *Algorithmic Cryptanalysis*, Cryptography and Network Security Series. Chapman and Hall/CRC, 2009.
10. Christopher Swenson. *Modern Cryptanalysis: Techniques for Advanced Code Breaking*. Wiley, 2008.
11. Jeffrey Hoffstien, Jill Pipher, and Joseph Silverman. *An Introduction to Mathematical Cryptography*. Springer, 2008.
12. Shafi Goldwasser and Mihir Bellare. *Lecture Notes on Cryptography*. Available at http://cseweb.ucsd.edu/~mihir/papers/gb.html, 2008.
13. Niels Ferguson. *Practical Cryptography*. Wiley, 2003.
14. Bruce Schneier. *Applied Cryptography: Protocols, Algorithms, and Source Code in C*. John Wiley \ Sons, Second Edition, 1996.

15. Albrecht Beutelspacher. *Cryptology*. Mathematical Association of America, 1994.

15 Technical Mathematics Area Assessment Outcomes

After completing the mathematics component of their general education, students in the College of Applied Science and Technology should possess the following competencies:

1. the ability to use algebraic tools when making qualitative and quantitative judgements;
2. the ability to recognize and solve technical problems using mathematical techniques;
3. the ability to recognize and use of a wide range of mathematical applications;
4. the ability to construct and interpret the various types of graphs;
5. an understanding the mathematical function concept and its applications;
6. the ability to use common technological tools when when solving mathematical problems;
7. the ability to utilize critical thinking and analytical skills when solving real-life problems;
8. knowledge of the effects of mathematics on human activities and society.