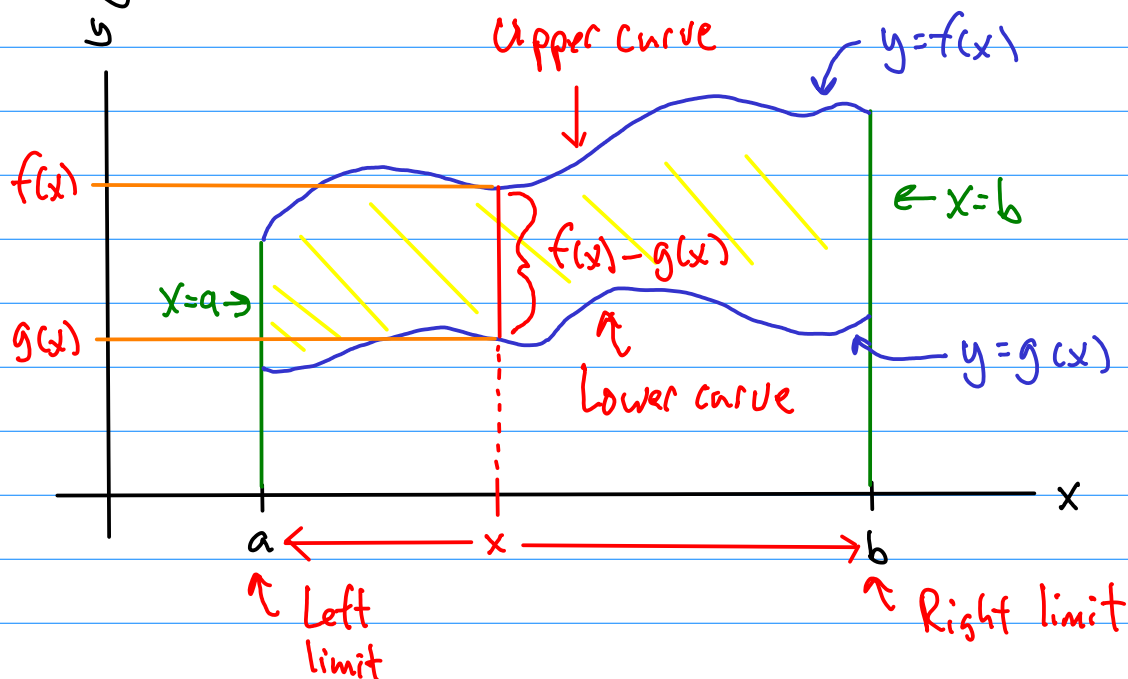


Area Between Curves

Suppose f and g are continuous functions on $[a, b]$ with antiderivatives F and G respectively. Suppose $f(x) > g(x)$ for all x in $[a, b]$.



The shaded region is bounded by $y=f(x)$, $y=g(x)$, $x=a$, and $x=b$.

$$\begin{aligned}
 \text{Area of shaded region} &= A \\
 &= \underbrace{\int_a^b f(x) dx}_{\substack{\text{Area under} \\ y=f(x) \text{ between} \\ x=a \text{ and } x=b}} - \underbrace{\int_a^b g(x) dx}_{\substack{\text{Area under} \\ y=g(x) \text{ between} \\ x=a \text{ and } x=b}} \\
 &= \int_a^b \underbrace{f(x) - g(x)}_{\substack{\text{Length of line segment} \\ \text{Left limit}}} dx \quad \text{Right limit} \quad x \text{ is changing}
 \end{aligned}$$

The area of the region is the sum of the lengths

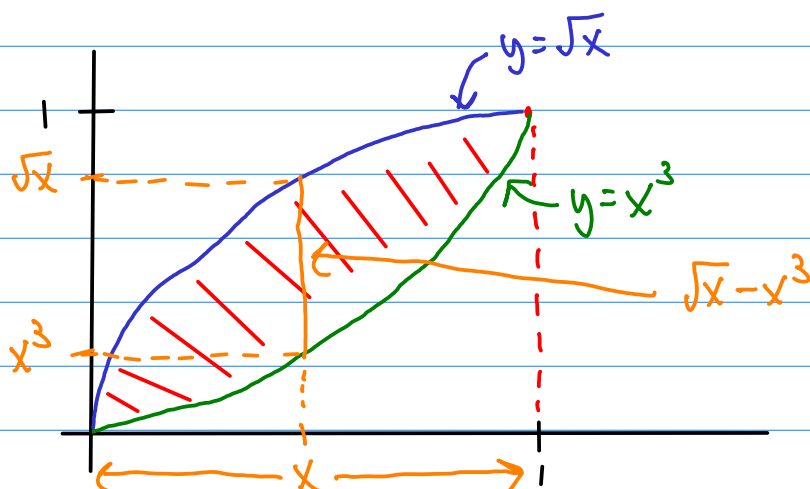
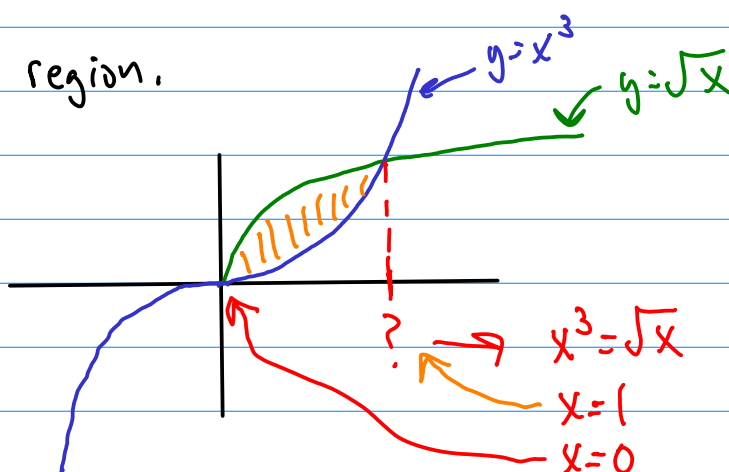
of all of the line segments. To find that sum we integrate.

$$A = \int_{\text{Left limit}}^{\text{Right limit}} (\text{Upper curve} - \text{Lower curve}) dx$$

Example: Find the area of the region bounded by $y = \sqrt{x}$ and $y = x^3$.

1) Sketch the region.

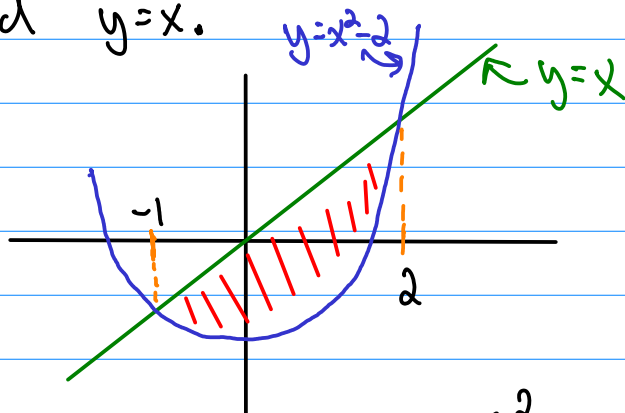
Rough



- 2) Find the area

$$\begin{aligned}
 A &= \int_0^1 \sqrt{x} - x^3 dx \\
 &= \int_0^1 x^{1/2} - x^3 dx = \left. \frac{2}{3} x^{3/2} - \frac{x^4}{4} \right|_0^1 \\
 &= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}
 \end{aligned}$$

- Example:** Find the area of the region bounded by $y = x^2 - 2$ and $y = x$.



$$\begin{aligned}
 x^2 - 2 &= x \\
 x^2 - x - 2 &= 0 \\
 (x+1)(x-2) &= 0 \\
 x &= -1, x = 2
 \end{aligned}$$

$$A = \int_{-1}^2 x - (x^2 - 2) dx = \int_{-1}^2 x - x^2 + 2 dx$$

$$= \left. \frac{x^2}{2} - \frac{x^3}{3} + 2x \right|_{-1}^2 = 2 - \frac{8}{3} + 4 - \left(\frac{1}{2} + \frac{1}{3} - 2 \right) = \frac{9}{2}$$